This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access

IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

*Digital Object Identifier 10.1109/ACCESS.2017.Doi Number*

# A Decision Method for Online Purchases Considering Dynamic Information Preference Based on Sentiment Orientation Classification and Discrete DIFWA Operators

**Zaoli Yang [1], Guanming Xiong [2], Zehong Cao [3], Yuchen Li[1], and Lucheng Huang[1]**

[1]College of Economics and Management, Beijing University of Technology, Beijing City 100124, China
[2]School of software and microelectronics, Peking University, Beijing City 102600, China
[3]Discipline of ICT, School of Technology, Environments and Design, College of Sciences and Engineering, University of Tasmania, TAS, Australia

Corresponding author: Yuchen Li (e-mail: liyuchen@ bjut.edu.cn).

This research was supported by the Natural Science Foundation of China under Grant 71704007 and the Beijing Social Science Foundation of China under Grant 18GLC082.

**ABSTRACT** Online reviews are crucial for evaluating product features and supporting consumers' purchase decisions. However, as a result of online buying behaviors, consumer habits and discrete dynamic distribution characteristics of online reviews, consumers typically randomly choose a limited number of reviews from discrete time frames among all reviews and give more weight to recent review information and less weight to earlier information to support their online purchase decisions; moreover, existing studies have ignored the discrete random dynamic characteristics and dynamic information preferences of consumers. To address this issue, this paper proposes a method based on sentiment orientation classification and discrete DIFWA (DDIFWA) operators for online purchase decisions considering dynamic information preferences. In this method, we transformed review texts from original discrete time slices to discrete random features, extracted product features based on the constructed feature and sentiment dictionaries and matched pairs of features and sentiment phrases in the dictionaries. We subsequently employed three types of semantic orientation by defining semantic rules to extract the product features of each review. Of note, the semantic orientations were transformed from discrete time to semantic intuitionistic fuzzy numbers and semantic intuitionistic fuzzy information matrixes. Furthermore, we proposed two DDIFWA operators to aggregate the dynamic semantic intuitionistic fuzzy information. Specifically, we obtained the rankings of alternative products and their features to support consumers' purchase decisions using the intuitionistic fuzzy scoring function and the "vertical projection distance" method. Finally, comparisons and experiments are provided to demonstrate the plausibility of our methods.

**INDEX TERMS** sentiment orientation classification, DDIFWA operator, dynamic information preference, online purchase decision

## I. INTRODUCTION

The success of e-commerce has led to a copious amount of customer reviews for online products and services. The reviews are posted on e-commerce platforms, media sites and forums, such as Tmall.com, JD.com, zol.com and mi.com. Online reviews have become a critical element for customers to compare products and make shopping decisions [1-3]. However, note that because the number of online reviews concerning the alternative products is typically large, it can be tedious and time-consuming for the consumer to read all of the online reviews one by one [2-3]. Thus, how to effectively rank products for a consumer's purchase decision through online reviews is a valuable research topic with extensive application backgrounds. In this situation, several methods can be helpful for the consumer to capture the information from the online reviews and make the purchase decision. These methods include identifying consumers' sentiment orientations by analyzing the online reviews [2-3], extracting a subset of important reviews [4], summarizing the opinions of a substantial number of online reviews [5] and

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access



determining the important product features or using a multicriteria decision method for ranking alternative products through online reviews [6-9]. In general, regardless of the sentiment analysis, extraction of important reviews, or determination of product features, the existing work has provided many ways to solve decision-making problems in a consumer's online purchase decision.

However, these methods fail to respond well to the problem of dynamic information aggregation, particularly without considering dynamic information preferences. Dynamic information aggregation is an important step in processing information and presenting decision results under a dynamic environment [10]. The online review information concerning a product recorded on the e-commerce platform is time dynamic. During the period from when the product is placed on shelves to when the product is taken off shelves, online reviews in different time periods have quite different impacts on consumer purchase decisions. The purpose of the existing studies [4-7, 11-13] is to rank similar products from the perspective of a third party. Thus, in most of the existing studies, the online review information for different time series is simply accumulated, without considering the consumers' subjective preferences for different time series information. Although the methods of [2-3, 8-9] aimed to provide purchase decision-making for consumers, which take into account subjective preferences of consumers, these methods aggregated product attribute information based on only consumer product attribute preferences and ignored the aggregation of dynamic review information from the time-series preference. In reality, consumers' online consumption behaviors and habits are characterized by more reliance on recent comments to support their purchase decisions, with less reliance on long-dated review information, and randomly reading a limited number of reviews from different discrete time periods to provide decision support for their online purchase. Thus, such selected review information demonstrates discrete random distribution features at the temporal level. In addition, for the same online product, not every time node has review information recorded in a continuous time period, and the review information of most products also presents discrete distribution features in the time-series dimension. Such review information discretely distributed in different time series exerts different influences on consumers' purchase decisions. Thus, continuous time-series weights and discrete time-series weights are quite different from each other. The latter can highlight consumers' greater preference differences for different time-series information than the former. In addition, many scholars focus on the idea of extracting consumer emotional orientation from the review information to solve product ranking and consumer purchase decision-making problems [4-7, 11-13]; however, these scholars analyzed only the positive or negative extreme emotional orientations of review opinions, while they neglected the neutral emotion orientation. Nevertheless, neutral emotion represents the uncertainty and hesitation of the review opinion and thus should not be ignored [2-3, 9]. On this basis, some scholars attempt to analyze the product ranking problem by combining intuitionistic fuzzy theory and sentiment analysis [2-3], which better integrated the three types of positive, negative and neutral emotional orientations. Although the previously described methods effectively aggregated emotional orientation information differently, the aggregation perspective is only from product attributes and ignored the time dynamic change perspective of emotion. By the same token, emotional information at different time series exerts different effects on consumers' decision-making. The active attenuation theory also indicates that it gives more weight to recent observations and less weight to earlier information with time passing [13-14], i.e., the thought of "stress the present rather than the past" [15]. Thus, to support the consumers' purchase decisions, it is necessary to develop an algorithm for extracting the review information under a discrete time-series information preference, providing a discrete dynamic information aggregation operator to aggregate dynamic review information under different time-series information preferences.

The objective of this paper is to propose a method based on the sentiment analysis technique and discrete dynamic intuitionistic fuzzy information aggregation operator to support consumer purchase decisions considering dynamic information preferences. The proposed method can be divided into two main parts, i.e., emotional orientation recognition of online reviews under different time series and product ranking based on discrete DIFWA (DDIFWA) operators. In the previous part, product review information is extracted from the JD Mall and Tmall Mall website, and the product feature dictionary and feature emotional orientation (including positive, negative, and neutral emotion orientations) dictionary are subsequently created based on the review information; moreover, three types of emotion orientation values for each feature of the product are calculated through an algorithm. In the latter part, from the time-series perspective, emotion information of reviews are selected from three discontinuous periods, the three emotional ratios of the same product feature in different time series are calculated, and the three types of emotional ratios are converted into emotion intuitionistic fuzzy numbers (IFNs) under multiple discrete time-series. At the same time, a DDIFWA operator is proposed to aggregate the emotion intuitionistic fuzzy evaluation matrix from the time-series preference. Finally, the intuitionistic fuzzy scoring function and the "vertical projection distance" method are used to evaluate online product features and rank the product.

The major contributions of this paper include the following three points:

(1) In terms of the online purchase decision problem, we first consider consumers' preferences for different time-series review information and highlight consumers' online consumption behavior features and habits in real life, which

coincide with the discrete dynamic distribution characteristics of online reviews, thus showing a very strong application background.

(2) In terms of the sentiment orientation classification, we used a self-built corpus to process word segmentation and part-of-speech (POS) tagging and established new specific feature and sentiment dictionaries, as well as optimized the sentiment identification based on the preliminary extraction of "feature word phrase–sentiment orientation word phrase" pairs. The accuracy of the model was improved by constructing "sentiment orientation word phrase–feature word phrase", "other word phrase" and "special sentiment word phrase" dictionaries.

(3) In terms of information aggregation, we discretized the review information for different time sequences, and we proposed and proved the DDIFWA operators to aggregate this discrete dynamic review information. Moreover, on the basis of the thought of "stress the present rather than the past", the discrete time weights are calculated using the time attenuation function.

The remainder of this paper is organized as follows. Section 2 introduces work related to our study. Section 3 describes data selection, data crawling, and data processing. Section 4 introduces sentiment orientation identification and classification. Section 5 provides the online product feature evaluation and purchase decision method based on sentiment IFNs. In Section 6, a case study is presented on the feature evaluation and ranking of five phones to illustrate the effectiveness of the presented method. The discussion is provided in Section 7 to verify the advantages of the method, and the conclusion and future work are provided in Section 8.

## II. RELATED WORK

In this section, we introduce several related studies in three areas: first, we investigate sentiment orientation classification; second, we consider online product ranking for consumers' purchase decisions; and third, we present information aggregation operators in IFNs.

### A. SENTIMENT ORIENTATION CLASSIFICATION

Sentiment orientation classification, also referred to as sentiment computing for online reviews, has received considerable attention in academia. The concept of sentiment computing was proposed by Picard from the Massachusetts Institute of Technology in 1997. In the book "Affective Computing", Picard [16] noted that sentiment computing is computing that relates to, arises from, or deliberately influences emotion or other affective phenomena. Sentiment orientation classification can be categorized into document level [17], statement level [18] or word level [19]. According to the literature [20-22], there are two thoughts for using the sentiment analysis technique to extract product features: dictionary-based extraction and corpus-based extraction. The first method involves using or extending a current sentiment dictionary or constructing a dictionary based on a corpus [20]. The most well-known English sentiment dictionaries are WordNet and SentiWordNet, and the most famous Chinese sentiment dictionaries are HowNet, NTUSD and Synonymy Thesaurus. In the first method, word phrases are crawled and filtered from an existing corpus and recorded in a dictionary. The sentiment orientation can subsequently be concluded by applying specific rules. Hu and Liu [21-22] were the first to develop the area of review mining. First, they conducted a POS analysis of reviews. Second, Hu's and Liu's groups [21-22] searched high-frequency word phrases to determine product features using a co-concurrence matrix. The sentiment orientation can then be inferred as positive, neutral or negative. The current classification methods for sentiment words primarily focus on autorecognition in the dictionary. Liu et al. [3] used HowNet to determine customers' sentiment orientation for car features [3]. Fu et al. [23] used topic modeling and the HowNet lexicon to analyze the aspects discussed in Chinese social reviews and the sentiments expressed. Turney and Littman [24] adopted the pointwise mutual information–information retrieval (PMI-IR) method to calculate the semantic orientation of word phrases. Asgarian et al. [25] developed a FerdowsNet and Persian sentiment lexicon for sentiment classification of the Persian language. Sentiment mining or classification has been extensively applied to analyze online reviews. Yuan et al. [26] extracted the consumer sentiment of product reviews using the joint sentiment-topic model to enhance sale prediction performance. Xiong et al. [27] proposed a joint sentiment-topic model, referred to as the word-pair sentiment-topic model (WSTM), for short text reviews and effectively applied the WSTM to mine product reviews. Fang et al. [28] proposed a multistrategy sentiment analysis method with semantic fuzziness to resolve the fuzziness of Chinese characters.

### B. ONLINE PRODUCT RANKING FOR CONSUMERS' PURCHASE DECISIONS

In terms of online product ranking, Liu's group [3] performed a comprehensive summary. They noted that there are few research papers that investigate overall rankings directly determined by online reviews. In particular, relevant studies [6-7, 11-13, 29] have focused on two directions: 1) determining product features and the overall sentiment orientation of customers towards a product by analyzing the online comments and 2) ranking products by the results of online review analysis. However, the previous studies have ignored the multidimensional complex sentiment orientation of product features, as well as their weighted impact on the overall rank. Moreover, the previous methods also ignored "neutral" as a sentiment orientation, thus leading to inaccurate and incomplete sentiment computation for the product features. Considering these gaps of neutral sentiment orientation and multidimensional complex sentiment orientation, Liu et al. proposed an algorithm based on sentiment dictionaries to identify positive, neutral and negative sentiment orientations for alternative products using the product features from automobilehome.com [3]. They subsequently transformed the positive, neutral and negative sentiment orientations into the membership, nonmembership and hesitancy degrees of IFNs, respectively, and constructed an online product ranking method based on a sentiment analysis technique and intuitionistic fuzzy set theory [3]. Fan et al. [8] presented a method for ranking products and supporting consumers' purchase decisions based on online product ratings and the attributes of each alternative product. Yang and Zhu [9] considered the large volume and normal distribution characteristics of online review information and proposed an online product purchasing decision method based on a normal stochastic multicriteria decision and the vertical distance method. Furthermore, scholars have analyzed the relationship between the features of online reviews and product demands or sales. Chong et al. [30] noted that online reviews and promotional marketing strategies are important predictors of product demands, and online reviews are generally better predictors than online marketing promotions. Ren et al. [31] noted that the volume of negative consumer reviews drives consumers' purchase decisions, whereas the volume of positive consumer reviews only marginally affects purchasing decisions. In particular, the helpfulness of an online review is an important online review feature, and a considerable amount of research has been conducted on the prediction, identification and influencing factors of online review helpfulness [32-34]. The existing research findings have optimized the online product ranking method; however, the data source of these methods is related product reviews, which overlook the fact that in their decision-making processes, consumers randomly read a limited number of online comments to support their purchase decision.

### C. INFORMATION AGGREGATION OPERATORS IN IFNS

Since Zadeh proposed fuzzy set theory in 1965 [35], many scholars have been expanding the fuzzy set and have proposed many new extension fuzzy sets, such as the neutrosophic set [36-37], hesitant fuzzy sets [38], Pythagorean fuzzy sets [39], probabilistic linguistic term sets[40], generalized fuzzy number[41], triangular fuzzy set [42], trapezoidal fuzzy set [43], intuitionistic fuzzy set [44] or Integrating Heterogeneous Information [45] , etc.. The intuitionistic fuzzy set is one of the important directions in the current development of fuzzy set theory. Referring to Zadeh's fuzzy set, the concept of the intuitionistic fuzzy set was proposed by Atanassov [44]. The intuitionistic fuzzy set, which can simultaneously express membership degree, nonmembership degree and hesitancy degree, can be applied to describe the fuzziness and uncertainty of objects. On this basis, many scholars have expanded upon the related research using the intuitionistic fuzzy set theory, particularly focusing on intuitionistic fuzzy aggregation operators and their application in decision making. For example, Xu [46-48] developed the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, the intuitionistic fuzzy hybrid aggregation (IFHA) operator, the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator. Wei [49] proposed an induced intuitionistic fuzzy ordered weighted geometric (I-IFOWG) operator and induced interval-valued intuitionistic fuzzy ordered weighted geometric (I-IIFOWG) operator, while Tan and Chen [50] proposed an intuitionistic fuzzy Choquet integral operator. Furthermore, Yu et al. [51] developed an interval-valued intuitionistic fuzzy prioritized operator, and Zhao and Wei [52] proposed intuitionistic fuzzy Einstein hybrid aggregation operators. Liu [53] proposed Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers. Joshi and Kumar [54] developed an interval-valued intuitionistic hesitant fuzzy Choquet integral operator. Liu [55] and Liu and Li [56] developed interval-valued intuitionistic fuzzy power Heronian aggregation operators and intuitionistic fuzzy interaction partitioned Bonferroni mean operators. Kaur and Garg [57] developed generalized cubic intuitionistic fuzzy aggregation operators, and Teng and Liu [58] proposed linguistic intuitionistic fuzzy density hybrid weighted averaging operators.

Scholars have also explored dynamic information aggregation with a time series. Xu and Yager [59] developed methods for weighting time series and proposed a dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator to resolve the issue of dynamic intuitionistic fuzzy multiattribute decision making. Wei [60] proposed a dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator and uncertain dynamic intuitionistic fuzzy weighted geometric (UDIFWG) operator. Park et al. [61] presented a dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator and uncertain dynamic intuitionistic fuzzy weighted geometric (UDIFWG) operator, while Yang et al. [15] presented the dynamic intuitionistic normal fuzzy weighted arithmetic average (DINFWAA) operator and the dynamic intuitionistic normal fuzzy weighted geometric average (DINFWGA) operator. Li et al. [61] developed dynamic interval-valued intuitionistic normal fuzzy aggregation operators, and Hao et al. [62] developed a dynamic decision-making approach by integrating prospect theory and an intuitionistic fuzzy Bayesian network to solve risk decision-making problems. Lourenzutti et al. [63] proposed two generalized approaches based on TOPSIS and TODIM to consider dynamic and heterogeneous information environments. Chen et al. [64] presented multiperiod power-weighted geometric average operators to address dynamic MADM problems in terms of an intuitionistic fuzzy environment. Yin et al. [65] proposed a dynamic intuitionistic fuzzy power geometric weighted average (DIFPGWA) operator. The previously described information aggregation operators can resolve the problem of aggregating dynamic information with continuous time; however, they cannot aggregate dynamic information with discrete time.

The previous studies have made significant contributions to sentiment orientation computing for online reviews, online product rankings and information aggregation operators. However, few studies have addressed sentiment orientation for each product feature, although the methods based on IFNs presented by Liu et al. [2] and Liu et al. [3] can reflect three types of sentiment orientations for product features, and the sentiment expression and the identification rules of product features are automatically obtained by the specific website. Thus, the sentiment orientation classification and product feature extraction of the existing methods cannot be widely applied to analyze online reviews of other e-commerce websites. Furthermore, the existing studies on dynamic information aggregation consider only continuous time cases while ignoring discrete time cases and consumers' information preferences, which make it quite difficult to describe the impact of a review from different time sequences on the consumer purchase decision. Thus, to support the consumers' purchase decisions, it is necessary to develop a method that can identify the sentiment orientation of each product feature, aggregate discrete dynamic information and rank alternative products according to the identified sentiment orientations and discrete dynamic information aggregation.

## III. DATA SELECTION, CRAWLING AND PROCESSIONG

### A. DATA SELECTION AND CRAWLING

Considering the scale of online shopping and the representativeness of mobile phone models, five mobile phone products, i.e., iPhone 8, Huawei Mate 10, Meizu PRO 7, Xiaomi Mi6 and VIVO X20, were selected as our research objects. Based on the Python platform, the crawler obtains online reviews from JD Mall and Tmall Mall. The time span for data crawling is from the release date of each mobile phone to May 5, 2018. The data source statistics are shown in Table 1.

**TABLE 1.** Data source statistics.

| Phone brand | Tmall Mall | | | Jingdong Mall | | | Total number of words |
|---|---|---|---|---|---|---|---|
| | Number of shops | Number of reviews | Number of words (ten thousand) | Number of shops | Number of reviews | Number of words (ten thousand) | |
| iPhone 8 | 103 | 26372 | 85.67 | 166 | 26228 | 79.05 | 164.72 |
| Mate 10 | 124 | 19838 | 73.38 | 147 | 15618 | 54.45 | 127.83 |
| Meizu PRO7 | 46 | 11432 | 42.93 | 55 | 3989 | 14.03 | 56.96 |
| Mi6 | 39 | 18352 | 63.41 | 41 | 11017 | 40.40 | 103.81 |
| VIVO X20 | 89 | 30125 | 121.99 | 26 | 12231 | 32.22 | 154.20 |
| Total number of words | -- | -- | 387.38 | -- | -- | 220.15 | 607.52 |

Table 1 shows that an original dataset of 6,075,200 words from online reviews for the 5 mobile phones is crawled, and a large number of duplicate reviews, invalid reviews, irregular characters and other data are cleaned. The cleaning rules include deleting control characters, switching traditional Chinese to simplified Chinese, replacing common typos, deleting repeated comments, deleting default comments and deleting specific characters.

### B. DATA PROCESSIONG

In this paper, the open-source Chinese word segmentation package jieba [67] is used for word segmentation and POS tagging. Product feature words (nouns) and corresponding sentiment words (adjectives) are identified by the system default segmentation dictionary and manual screening.

First, the default word segmentation dictionary of jieba is used to perform preliminary word segmentation, extract nouns and adjectives according to the word segmentation results, count the frequency of each word, select words with higher frequencies, and classify the words to create feature word and sentiment dictionaries. In terms of feature words, 11,232 nouns are generated, and considering the word frequency distribution, a long tail characteristic is presented. The 482 words with the highest frequency are selected to create a feature dictionary. The cumulative word frequency is approximately 85%. The rules for creating sentiment dictionaries are similar. There are 1,709 sentiment words, and the 117 most frequently used words are selected as sentiment dictionary sets. The cumulative frequency is approximately 85%. The dictionary in the first step is subsequently used for highlight matching, and the unidentified words are manually screened and added to the dictionary. The product feature dictionary and the product sentiment dictionary are then created by the following process: previously described dictionary → dictionary trial matching → highlight check → manual screening.

## IV. SENTIMENT ORIENTATION CLASSIFICATION

The sentiment analysis method adopted in this paper is based on dictionary matching feature-sentiment word pairs and

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access



constructing semantic rules to complete the sentiment orientation classification.

The first step is to identify the nouns (feature words). First, a new dictionary is created and used for word segmentation and POS tagging. After a noun is found, the feature to which it corresponds is evaluated. For example, "做工 (workmanship)" and "质量 (quality)" can be classified as quality feature words. The specific rules are shown in Table 2.

**TABLE 2.** **Rules of POS tagging.**

| Feature | Feature word | Sentiment orientation | Sentiment word | Negation words |
|---|---|---|---|---|
| Quality | {做工(workmanship) n_ZL}, {品质 (character) n_ZL}…… | positive | #艳丽(marvelous) GD_adj, #优秀(excellent) GD_adj…… | |
| Appearance | {手感(sense of touch)n_WG}, {效果 (effect) n_WG}…… | | | |
| System | {IOSn_XT}, {Android n_XT}…… | neutral | #一般(not bad) NON_adj, #还行(ordinary) NON_adj…… | #不是(not) FDC, #没有(no) FDC…… |
| Configuration | {处理器(cpu) n_PZ}, {容量(capacity) n_PZ}…… | | | |
| Price | {性价比(cost performance) n_JG}, {cost () n_PP}…… | negative | #差劲(poor) NEG_adj, #坑人(deceptive) NEG_adj…… | |
| Brand | {国产机(China mobile phone) n_PP}, {Apple n_PP}…… | | | |

The second step is to identify the sentiment words (adjectives). The sentiment words are divided into three categories: 1) positive sentiment; 2) neutral sentiment; and 3) negative sentiment. For example, "marvelous" and "excellent" are positive sentiments; "not too bad" and "ordinary" are neutral sentiments; and "poor" and "deceptive" are negative sentiments. The specific rules are shown in Table 2.

The third step is to identify negation words. In general, "not", "no", etc., are marked as negation words following the specific rules shown in Table 2.

The fourth step is to create a regular expression for sentiment orientation identification. Based on the POS tagging rules shown in Table 2, the regular expression rules are shown in Table 3.

**TABLE 3.** **Regular expression of sentiment orientation identification, taking "质量 (quality)" as an example.**

| Feature | Sentiment orientation | Regular expression |
|---|---|---|
| Quality | positive | {[^{]*?n_ZL}[^#]*?#[^#]*?GD_adj<br>{[^{]*?n_ZL}[^#]*?#FDC[^#]*?GD_adj |
| | neutral | {[^{]*?n_ZL}[^#]*?#[^#]*?NON_adj |
| | negative | {[^{]*?n_ZL}[^#]*?#[^#]*?NEG_adj<br>{[^{]*?n_ZL}[^#]*?#FDC[^#]*?NEG _adj |

Taking the "quality" feature as an example, the sentiment orientation of each feature is divided into three directions: positive, neutral and negative. The regular expressions are created based on sentiment identification. According to the semantic rules, positive sentiments have two representation types: 1) feature words–positive sentiment word pairs (i.e., nouns + adjectives) and 2) feature words–negation words–negation sentiment word pairs. Similarly, negative sentiments can be expressed using feature words–negation sentiment word pairs and feature words–negation words—positive sentiment word pairs. The neutral sentiment orientation has only one identification pattern, which are feature words–neutral sentimental word pairs.

The fifth step is to optimize the results of sentiment orientation identification. The optimization process primarily includes the following three aspects.

(1) Implicit feature identification is applied based on PMI. In response to the colloquial phenomenon of product reviews, many reviews are expressed not only in the previously described mode of "feature words–sentiment words" but also in pure adjective form because a single adjective conveys two dimensions of information, namely, features and sentiments. Therefore, the PMI value calculation method based on an Internet search engine is used to identify the implicit feature [68]. In this method, the search results are used as the source of word probability. The decrement PMI threshold method [69] is subsequently used with a threshold x=40%. The PMI is arranged in descending order to evaluate the sentimental orientation of the feature words. Moreover, the semantic orientation PMI (SO-PMI) [70] algorithm is introduced to calculate the sentiment orientation of sentiment words.

(2) The combination of one adjective with different feature words may produce different sentimental orientations. For example, "very fast start" is a positive evaluation, while "very fast power consumption" is a negative evaluation. For these identification problems, the manual annotation method is adopted in this paper, and a new dictionary of special words is created, as shown in Table 4.

**TABLE 4. New dictionary of special words (application example).**

| Feature | Words | Marking | Regular expression (negative) |
|---|---|---|---|
| Quality | 缝隙(crack)<br>耗电(power consumption)<br>划痕(scratch) | N_ZL_NEG | {[^{]*?n_ZL_NEG}[^#]*?#[^#]*?GD_adj<br>#[^#]*?GD_adj[^#]*? {[^{]*?n_ZL_NEG} |
| Appearance | 阴阳屏(screen of Yin and Yang) | N_WG_NEG | {[^{]*? N_WG_NEG}[^#]*?#[^#]*?NON_adj<br>#[^#]*?GD_adj[^#]*? {[^{]*?n_ZL_NEG} |

(3) An additional feature dictionary is employed. Some feature words are mismatched to adjectives that describe other features. For example, "mobile phone" and "logistics" can be recognized as nouns in "mobile phone is still used, logistics are very satisfactory"; however, because "logistics" does not originate from a specific dictionary, its matching adjective, satisfactory, is falsely matched with mobile phone, and the result is "mobile phone... satisfactory", which is not ideal. Therefore, the proposed solution is to add a "miscellaneous" feature dictionary to classify nontarget feature words and establish corresponding matching patterns.

Finally, precision, recall and f-score are used to measure the accuracy of sentiment orientation classification. In this section, 100 reviews are randomly selected for each mobile phone as test sets for manual testing. The test results show that precision, recall and f-score increase from 67.4%, 38.6% and 51.8%, respectively, before optimization to 81.4%, 66.8% and 73.3%, respectively, after optimization.

## V. PRODUCTS RANKING FOR PURCHASE DECISION METHOD BASED ON SENTIMENT INTUITIONSTIC FUZZY NUMBERS

### A. INTUITIONSTIC FUZZY NUMBERS

Definition **1** [44]. Let $X$ be a fixed set, where $X=(x_1,x_2,\cdots,x_m)$. $A=\left\{\left[x,\mu_A(x),\upsilon_A(x)\right] \middle| x\in X\right\}$ is an intuitionistic fuzzy set $A$ in $X$; $\mu_A(x)$ and $\upsilon_A(x)$ denote the membership and nonmembership of element $x$ in $X$ to $A$, respectively; $\mu_A(x):X\to[0,1]$; $\upsilon_A(x):X\to[0,1]$; $0\le\mu_A(x)+\upsilon_A(x)\le 1, x\in X$; and $\pi_A=1-\mu_A(x)-\upsilon_A(x)$ is the hesitancy degree of $x$ in $X$ to $A$. The ordered pair $\alpha=<\mu_\alpha,\upsilon_\alpha>$, which is composed of membership and nonmembership, is referred to as an IFN.

Definition **2** [44]. Let $a=<\mu_a,\upsilon_a>$ be an IFN. $S(a)=\mu_a-\upsilon_a$ and $H(a)=\mu_a+\upsilon_a$ are defined as the score function and accuracy function, respectively. For IFNs $a=<\mu_a,\upsilon_a>$ and $b=<\mu_b,\upsilon_b>$, $S(a)=\mu_a-\upsilon_a$, $H(a)=\mu_a+\upsilon_a$, $S(b)=\mu_b-\upsilon_b$, and $H(b)=\mu_b+\upsilon_b$ are the score degrees and accuracy degrees. The ranking rule for IFNs is shown below:

1) If $S(a)>S(b)$, then IFN $a$ is larger than IFN $b$.

2) If $S(a)=S(b)$ and

- if $H(a)=H(b)$, then IFN $a$ is equal to IFN $b$;
- if $H(a)<H(b)$, then IFN $a$ is smaller than IFN $b$;
- if $H(a)>H(b)$, then IFN $a$ is larger than IFN $b$.

Definition **3** [44]. Let $\alpha=<\mu_\alpha,\upsilon_\alpha>$, $\alpha_1=<\mu_{\alpha_1},\upsilon_{\alpha_1}>$ and $\alpha_2=<\mu_{\alpha_2},\upsilon_{\alpha_2}>$ be three IFNs. The basic operational laws are summarized as follows:

1) $\alpha_1\oplus\alpha_2=<\mu_{\alpha_1}+\mu_{\alpha_2}-\mu_{\alpha_1}\mu_{\alpha_2},\upsilon_{\alpha_1}\upsilon_{\alpha_2}>$;

2) $\alpha_1\otimes\alpha_2=<\mu_{\alpha_1}\mu_{\alpha_2},\upsilon_{\alpha_1}+\upsilon_{\alpha_2}-\upsilon_{\alpha_1}\upsilon_{\alpha_2}>$;

3) $\lambda\alpha=<1-(1-\mu_\alpha)^\lambda,\upsilon_\alpha{}^\lambda>$;

4) $\alpha^\lambda=<\mu_\alpha{}^\lambda,1-(1-\upsilon_\alpha)^\lambda>$.

### B. SENTIMENT INTUITIONISTIC FUZZY NUMBERS

#### 1) SENTIMENT ORIENTATION CLASSIFICATION OF PRODUCT FEATURES

A consumer has a total of $n$ choices for products and discretely and randomly reads the reviews under limited $k$ time sequences of the whole time $T$. Assume that the selection set of products in the $t_\zeta$-th discrete time sequence is:

$$A^{t_\zeta}=\left\{A_1^{t_\zeta},A_2^{t_\zeta},\cdots,A_n^{t_\zeta}\right\} \quad (1)$$

where $t_\zeta=a,f,o,\cdots,z\in Z^+$, and $t_\zeta$ is a discontinuous time sequence; that is, $f\ne a+1$ or $o\ne f+1$. Moreover, $\zeta=1,2,\cdots,k$ is a numbering matched the sequence of $k$ discrete time; that is, $t_1=a$, $t_2=f$,……, $t_k=z$.

The evaluation index set of each product is feature set $C$, where there are a total of $m$ features. Then:

$$C^{t_\zeta}=\left\{C_1^{t_\zeta},C_2^{t_\zeta},\cdots,C_m^{t_\zeta}\right\}$$

The weight vector of attributes under the $t_\zeta$-th discrete time is $w^{t_\zeta}=(w_1^{t_\zeta},w_2^{t_\zeta},\cdots,w_m^{t_\zeta})^T$.

Let the $i$-th product $A_i^{t_\zeta},(i=1,2,\cdots,n)$ correspond to $q_i^{t_\zeta}$ reviews in the $t_\zeta$-th discrete time sequence, and $Q^{t_\zeta}=\left\{q_1^{t_\zeta},q_2^{t_\zeta},\cdots,q_n^{t_\zeta}\right\}$ indicates the vector of numbers of the online reviews of $n$ products; then, the $l$-th review concerning alternative phone $A_i^{t_\zeta}$ under the $t_\zeta$-th discrete time recorded as $D_i^{lt_\zeta}=(D_{i1}^{lt_\zeta},D_{i2}^{lt_\zeta},\cdots,D_{im}^{lt_\zeta}),(l=1,2,\cdots,q_i^{t_\zeta})$, where

$D_{ij}^{lt_\zeta},(j=1,2,\cdots,m)$ denotes the sentence on the attribute $C_{ij}^{t_\zeta},(j=1,2,\cdots,m)$ of product $A_i^{t_\zeta}$.

Since the number of sentiment orientations of online reviews in each discrete time slice stage is large, it is necessary to aggregate the comment information on the same sentiment orientation of a certain product feature to obtain comprehensive comment information. To this end, the weight of each comment on a product feature is required, and the number of the same sentiment orientations for a certain product feature is multiplied by the online review weight to derive a comprehensive product feature-sentiment orientation. According to Najmi [13], the active attenuation index derived from biology can effectively measure the importance of online comments. Najmi [13] noted that over time, the impact of consumers' comments will show exponential decay. Therefore, the time exponential decay method is used to obtain the weight of each online comment in this paper as follows.

Let $\omega_{ij}^{lt_\zeta},(i=1,2,\cdots,n;\ j=1,2,\cdots,m)$ be the weight of the online review $D_{ij}^{lt_\zeta}$ in the $t_\zeta$-th discrete time. The formula for $\omega_{ij}^{lt_\zeta}$ is:

$$\omega_{ij}^{lt_\zeta}=e^{\frac{T_{ij}^{lt_\zeta}-T_i^{t_\zeta}}{T_c^{t_\zeta}-T_i^{t_\zeta}}},(i=1,2,\cdots,n;\ j=1,2,\cdots,m;l=1,2,\cdots,q_i^{t_\zeta}) \tag{2}$$

where $T_{ij}^{lt_\zeta}$ represents the release time of product $A_i^{t_\zeta}$ under the $t_\zeta$-th discrete time stage, $T_i^{t_\zeta}$ represents the shelf time of product $A_i^{t_\zeta}$, and $T_c^{t_\zeta}$ represents the current time. We can see that for the exponential term of the formula, the numerator is always less than or equal to the denominator; thus, the numerator/denominator is less than or equal to 1, and the total weight is greater than 0 and less than or equal to $e$.

Therefore, we can obtain the weighted value of each product feature sentimental orientation in $k$ different discrete time sequences:

$$POS_{ij}^{t_\zeta}=\sum_{l=1}^{q_i^{t_\zeta}}\left(\omega_{ij}^{lt_\zeta}*count\_pos_{ij}^{lt_\zeta}\right) \tag{3}$$
$$(i=1,2,\cdots,n;\ j=1,2,\cdots,m;l=1,2,\cdots,q_i^{t_\zeta})$$

$$HES_{ij}^{t_\zeta}=\sum_{l=1}^{q_i^{t_\zeta}}\left(\omega_{ij}^{lt_\zeta}*count\_hes_{ij}^{lt_\zeta}\right) \tag{4}$$
$$(i=1,2,\cdots,n;\ j=1,2,\cdots,m;l=1,2,\cdots,q_i^{t_\zeta})$$

$$NEG_{ij}^{t_\zeta}=\sum_{l=1}^{q_i^{t_\zeta}}\left(\omega_{ij}^{lt_\zeta}*count\_neg_{ij}^{lt_\zeta}\right) \tag{5}$$
$$(i=1,2,\cdots,n;\ j=1,2,\cdots,m;l=1,2,\cdots,q_i^{t_\zeta})$$

In formula (3), $POS_{ij}^{t_\zeta}$ represents a summary of positive sentiment values for the $j$-th feature of the $i$-th mobile phone at the $t_\zeta$-th discrete time stage, $count\_pos_{ij}^{lt_\zeta}$ represents the number of the $j$-th feature positive sentiment in the $l$-th comment of the $i$-th mobile phone, and $\omega_{ij}^{lt_\zeta}$ is the weight of the $l$-th online review. HES and NEG represent a summary of neutral and negative sentiment values, respectively, and the remaining symbols have similar meanings. Therefore, a database is provided for the next step to construct a sentimental intuitionistic fuzzy evaluation matrix based on three types of product feature-sentiment orientations.

### 2) SENTIMENT INTUITIONISTIC FUZZY EVALUATION MATRIX FOR A DISCRETE TIME SEQUENCE

According to the previously described analysis, the online review sentimental orientations of each product feature consist of three types: positive, neutral, and negative. According to Liu et al. [2] and Liu et al. [3], the three sentiment orientations are normalized, and the proportion of each sentiment orientation among all sentiments is calculated. The proportions correspond to membership degree $\mu_\alpha$, nonmembership degree $\upsilon_\alpha$ and hesitancy degree $1-\mu_\alpha-\upsilon_\alpha$ in the IFN under the $t_\zeta$-th discrete time as follows:

$$\begin{aligned}\bar{R}&=\left(\bar{r}_{ij}^{t_\zeta}\right)_{n\times m}=\left(<\bar{\mu}_{ij}^{t_\zeta},\bar{\upsilon}_{ij}^{t_\zeta}>\right)_{n\times m}\\&=\left(<\frac{POS_{ij}^{t_\zeta}}{POS_{ij}^{t_\zeta}+HES_{ij}^{t_\zeta}+NEG_{ij}^{t_\zeta}},\frac{HES_{ij}^{t_\zeta}}{POS_{ij}^{t_\zeta}+HES_{ij}^{t_\zeta}+NEG_{ij}^{t_\zeta}}>\right)_{n\times m}\end{aligned} \tag{6}$$

From this, we obtain $k$ matrixes (each matrix is of size $n\times m$), and each matrix element is a pair of sentiment IFNs. The matrix is developed as follows:

$$\begin{aligned}\bar{R}^{t_\zeta}&=\left(\bar{r}_{ij}^{t_\zeta}\right)_{n\times m}=\left(<\bar{\mu}_{ij}^{t_\zeta},\bar{\upsilon}_{ij}^{t_\zeta}>\right)_{n\times m}\\&=\begin{bmatrix}<\bar{\mu}_{11}^{t_\zeta},\bar{\upsilon}_{11}^{t_\zeta}> & <\bar{\mu}_{12}^{t_\zeta},\bar{\upsilon}_{12}^{t_\zeta}> & \cdots & <\bar{\mu}_{1m}^{t_\zeta},\bar{\upsilon}_{1m}^{t_\zeta}>\\<\bar{\mu}_{21}^{t_\zeta},\bar{\upsilon}_{21}^{t_\zeta}> & <\bar{\mu}_{22}^{t_\zeta},\bar{\upsilon}_{22}^{t_\zeta}> & \cdots & <\bar{\mu}_{2m}^{t_\zeta},\bar{\upsilon}_{2m}^{t_\zeta}>\\\vdots & \vdots & \ddots & \vdots\\<\bar{\mu}_{n1}^{t_\zeta},\bar{\upsilon}_{n1}^{t_\zeta}> & <\bar{\mu}_{n2}^{t_\zeta},\bar{\upsilon}_{n2}^{t_\zeta}> & \cdots & <\bar{\mu}_{nm}^{t_\zeta},\bar{\upsilon}_{nm}^{t_\zeta}>\end{bmatrix}\end{aligned} \tag{7}$$

where $t_\zeta$ is a discontinuous discrete time sequence and $t_\zeta=a,f,o,\cdots,z$. To aggregate the sentiment intuitionistic fuzzy matrix information for different discrete time sequences, it is necessary to find the weight and the information aggregating operator of the discrete time sequence.

In addition, considering the difference in product feature weights for different discrete time sequences, in this paper, the intuitionistic fuzzy entropy (IFE) [71] is used to obtain the product feature weight $w^{t_\zeta}=(w_1^{t_\zeta},w_2^{t_\zeta},\cdots,w_m^{t_\zeta})^T$, and we can also obtain the feature-weighted sentiment intuitionistic fuzzy matrix as follows:

$$R^{t_\zeta} = \left(< \mu_{ij}^{t_\zeta}, \upsilon_{ij}^{t_\zeta} >\right)_{n\times m} = \bar{R}^{t_\zeta} \times w^{t_\zeta} = \left(< 1-(1-\bar{\mu}_{ij}^{t_\zeta})^{w_j^{t_\zeta}}, (\bar{\upsilon}_{ij}^{t_\zeta})^{w_j^{t_\zeta}} >\right)_{n\times m} \tag{8}$$

In Eq. (8), $w_j^{t_\zeta}$ is the weight of the $j$-th feature of the $i$-th mobile phone in the $t_\zeta$-th discrete time sequence.

#### 3) ESTABLISHMENT OF THE DISCRETE TIME WEIGHT AND THE DDIFWA OPERATOR

A key premise of the aggregation of discrete time information is a determination of the time weight. At the same time, considering the trend of consumers' purchasing decisions, consumers focus on the timeline of product review information; that is, the more recent the information, the more valuable it is to the consumer’s decision making. To this end, the time attenuation function in reference [13] is introduced to determine the discrete time weights. In contrast to the continuous time weight provided by Eq. (2), the time weights under a discrete time sequence are provided here, and the formula is derived as follows:

Let $t_\zeta = a, f, o, \cdots, z\, (\zeta = 1, 2, \cdots, k)$ denote a discrete time sequence where the weight of time $t_\zeta$ is $\eta(t_\zeta)$; then,

$$\eta(t_\zeta) = C_0 e^{\lambda(t_\zeta - t_1)}, (t_\zeta = a, f, o, \cdots, z; \zeta = 1, 2, \cdots, k) \tag{9}$$

In Eq. (9), $C_0 > 0$ is a constant in the discrete time sequence, and $0 < \lambda < 1$ is an attenuation coefficient.

Similarly, according to the constraint condition of the weight, the discrete time weight $\eta(t_\zeta)$ should satisfy $\sum_{\zeta=1}^{k} \eta(t_\zeta) = \sum_{t_\zeta=t_1=a}^{t_k=z} C_0 e^{\lambda(t_\zeta - t_1)} = 1$. Thus, $C_0 = 1 \Big/ \sum_{t_\zeta=t_1=a}^{t_k=z} C_0 e^{\lambda(t_\zeta - t_1)}$ can be obtained, and Eq. (9) can be converted into:

$$\eta(t_\zeta) = \frac{e^{\lambda(t_\zeta - t_1)}}{\sum_{t_\zeta=t_1=a}^{t_k=z} C_0 e^{\lambda(t_\zeta - t_1)}} = \frac{e^{\lambda t_\zeta}}{\sum_{t_\zeta=t_1=a}^{t_k=z} C_0 e^{\lambda t_\zeta}} \tag{10}$$

$$(t_\zeta = a, f, o, \cdots, z; \zeta = 1, 2, \cdots, k)$$

In Eq. (10), $\eta(t_\zeta)$ is the weight of $t_\zeta$, and $0 < \lambda < 1$ is an attenuation coefficient. $t_\zeta = a, f, o, \cdots, z$ indicates that the t ime intervals are unequal, which is fundamentally different from the time weight determination method for a traditional continuous time sequence.

After obtaining the discrete time weights, it is necessary to aggregate the discrete time sequence information using the information aggregation operator. To this end, the DDIFWA operators are proposed, including the discrete dynamic intuitionistic fuzzy average weighted aggregation (DDIFWAA) operator and the discrete dynamic intuitionistic fuzzy geometric average weighted aggregation (DDIFWGAA) operator. The definitions are as follows:

Let $t_\zeta$ be a time sequence variable; then, $a(t_\zeta) =< \mu_{(t_\zeta)}, \upsilon_{(t_\zeta)} >$ is called an IFN, where $\mu_{a(t_\zeta)} \in [0,1], \upsilon_{a(t_\zeta)} \in [0,1]$ and $\mu_{a(t_\zeta)} + \upsilon_{a(t_\zeta)} \le 1$. If $t_\zeta = a, f, o, \cdots, z\, (\zeta = 1, 2, \cdots, k)$, then $a(a), a(f), a(o) \cdots, a(z)$ represent the IFNs of $k$ different discrete time sequences.

**Theorem 1.** Let $a(t) = (a(t_1), a(t_2), a(t_3), \cdots, a(t_k)) = (a(a), a(f), a(o), \cdots, a(z))$ be the IFNs of the $k$ time sequences and $\eta(t_\zeta) = (\eta(t_1), \eta(t_2), \eta(t_3), \cdots, \eta(t_k))^T = (\eta(a), \eta(f), \eta(o), \cdots, \eta(z))^T$ represent discrete time weight vector, $\eta(t_\zeta) \in [0,1]$, $\sum_{t_\zeta=t_1=a}^{t_k=z} \eta(t_\zeta) = 1$. Let $DDIFWAA: Q^n \to Q$, then:

$$DDIFWAA_{\eta(t_\zeta)}(a(a), a(f), a(o), \cdots, a(z)) = \sum_{t_\zeta=t_1=a}^{t_k=z} \eta(t_\zeta) \times a(t_\zeta) = (1 - \prod_{t_\zeta=t_1=a}^{t_k=z} (1-\mu_{(t_\zeta)})^{\eta(t_\zeta)}, \prod_{t_\zeta=t_1=a}^{t_k=z} \upsilon_{(t_\zeta)}{}^{\eta(t_\zeta)}) \tag{11}$$

is a DDIFWAA operator.

**Theorem 2.** Let $a(t) = (a(t_1), a(t_2), a(t_3), \cdots, a(t_k)) = (a(a), a(f), a(o), \cdots, a(z))$ be the IFNs of the $k$ time sequences, $\eta(t_\zeta) = (\eta(t_1), \eta(t_2), \eta(t_3), \cdots, \eta(t_k))^T = (\eta(a), \eta(f), \eta(o), \cdots, \eta(z))^T$ represent discrete time weight vector, $\eta(t_\zeta) \in [0,1]$, $\sum_{t_\zeta=t_1=a}^{t_k=z} \eta(t_\zeta) = 1$. Let $DDIFWGAA: Q^n \to Q$, then:

$$DDIFWGAA_{\eta(t_\zeta)}(a(a), a(f), a(o), \cdots, a(z)) = \prod_{t_\zeta=t_1=a}^{t_k=z} \eta(t_\zeta) \times a(t_\zeta) = (\prod_{t_\zeta=t_1=a}^{t_k=z} \mu_{(t_\zeta)}{}^{\eta(t_\zeta)}, 1 - \prod_{t_\zeta=t_1=a}^{t_k=z} (1-\upsilon_{(t_\zeta)})^{\eta(t_\zeta)}) \tag{12}$$

is a DDIFWGAA operator.

The proofs of Theorem 1 and 2 are provided in Appendix.

According to the discrete time weights, the DDIFWA operators are used to aggregate the information for different time sequences to obtain the discrete dynamic intuitionistic fuzzy synthesis decision matrix (DDIFSDM).

The DDIFSDM based on the DDIFWAA operator is as follows:

$$R = \left(< \mu_{ij}, \upsilon_{ij} >\right)_{n\times m} = \sum_{\zeta=1}^{k} R^{t_\zeta} \times \eta(t_\zeta) = \left(1 - \prod_{\zeta=1}^{k} (1-\mu_{ij(t_\zeta)})^{\eta(t_\zeta)}, \prod_{\zeta=1}^{k} \upsilon_{ij(t_\zeta)}{}^{\eta(t_\zeta)}\right)_{n\times m} \tag{13}$$

The DDIFSDM based on the DDIFWGAA operator is as follows:

$$R=\left(<\mu_{ij},\upsilon_{ij}>\right)_{n\times m}=\prod_{\zeta=1}^{k}R^{t_\zeta\,\eta(t_\zeta)}$$
$$=\left(\prod_{\zeta=1}^{k}\mu_{ij(t_\zeta)}{}^{\eta(t_\zeta)},1-\prod_{\zeta=1}^{k}(1-\upsilon_{ij(t_\zeta)})^{\eta(t_\zeta)}\right)_{n\times m} \tag{14}$$

### C. ONLINE PRODUCT FEATURES AND COMPREHENSIVE RANKING

Based on the DDIFSDM, the intuitionistic fuzzy score $S(a)$ and accuracy $H(a)$ in Definition 2 are used to rank the features of all alternative products.

For a comprehensive product ranking, the vertical projection distance method reported by Yang [9] is used to sort the synthesis scores of each alternative product. The vertical distance algorithm is described as follows.

First, the positive and negative ideal solutions of alternatives in the DDIFSDM $R$ are determined. The positive ideal solution is as follows:

$$r^+=(<\mu_1^+,\upsilon_1^+,\pi_1^+>,<\mu_2^+,\upsilon_2^+,\pi_2^+>,\cdots,<\mu_m^+,\upsilon_m^+,\pi_m^+>) \tag{15}$$

where $\mu_j^+=\max_i\{\mu_{ij}\}$, $\upsilon_j^+=\min_i\{\upsilon_{ij}\}$, and $\pi_j^+=1-\mu_j^+-\upsilon_j^+$.

The negative ideal solution is as follows:

$$r^-=(<\mu_1^-,\upsilon_1^-,\pi_1^->,<\mu_2^-,\upsilon_2^-,\pi_2^->,\cdots,<\mu_m^-,\upsilon_m^-,\pi_m^->) \tag{16}$$

where $\mu_j^-=\min_i\{\mu_{ij}\}$, $\upsilon_j^-=\max_i\{\upsilon_{ij}\}$, and $\pi_j^+=1-\mu_j^+-\upsilon_j^+$.

Then, the linear distance between each alternative and the positive or negative ideal solutions and the linear distance between the positive and negative ideal solutions are obtained:

$$d(r_i,r^+)=\sqrt{\frac{1}{2}\sum_{j=1}^{m}\left[(\mu_j^+-\mu_{ij})^2+(\upsilon_j^+-\upsilon_{ij})^2+(\pi_j^+-\pi_{ij})^2\right]} \tag{17}$$

$$d(r_i,r^-)=\sqrt{\frac{1}{2}\sum_{j=1}^{m}\left[(\mu_j^--\mu_{ij})^2+(\upsilon_j^--\upsilon_{ij})^2+(\pi_j^--\pi_{ij})^2\right]} \tag{18}$$

$$d(r^+{}_i,r^-)=\sqrt{\frac{1}{2}\sum_{j=1}^{m}\left[(\mu_j^--\mu_j^+)^2+(\upsilon_j^--\upsilon_j^+)^2+(\pi_j^--\pi_j^+)^2\right]} \tag{19}$$

Finally, the "vertical projection" distance between each alternative and the positive ideal solution is identified as follows:

$$V_i^+=\frac{d^2(r^+,r^-)+d^2(r^+,r_i)-d^2(r^-,r_i)}{2d(r^+,r^-)} \tag{20}$$

In a similar way, the "vertical projection" distance between each alternative and the negative ideal solution is identified as follows:

$$V_i^-=\frac{d^2(r^+,r^-)+d^2(r^-,r_i)-d^2(r^+,r_i)}{2d(r^+,r^-)} \tag{21}$$

The smaller the value of $V_i^+$, the better the alternative product, and vice versa. Moreover, the larger the value of $V_i^-$ is, the better the alternative product, and vice versa.

### D. ALGORITHM OF THE PROPOSED METHOD

Given the previously described analysis, we proposed an online purchase decision method based on sentiment orientation classification and the DDIFWA operator. The algorithm was developed as follows.

**Step 1** The alternative products and data sources are identified, and data based on the Python platform are processed to create dictionaries of product features and their sentiment orientation words.

**Step 2** The time slice basis of the discrete time sequence is determined, and online reviews of $k$ discrete time are extracted to construct the vector of numbers of the online reviews $Q^{t_\zeta}=\left\{q_1^{t_\zeta},q_2^{t_\zeta},\cdots,q_n^{t_\zeta}\right\}$ of $n$ products and the vector of the $l$ -th review $D_i^{lt_\zeta}=(D_{i1}^{lt_\zeta},D_{i2}^{lt_\zeta},\cdots,D_{im}^{lt_\zeta}),(l=1,2,\cdots,q_i^{t_\zeta})$ concerning alternative product $A_i^{t_\zeta}$.

**Step 3** The sentiment orientation of each product feature is calculated, and the time exponential decay method for a continuous time sequence is subsequently used to weight and aggregate the same feature sentiments in different reviews using Eq. (2). The three kinds of sentiments for each product feature are obtained, i.e., $POS_{ij}^{t_\zeta}$, $HES_{ij}^{t_\zeta}$ and $NEG_{ij}^{t_\zeta}$, using Eq. (4)–Eq. (6), respectively.

**Step 4** According to the three kinds of feature sentiments, the sentimental intuitionistic fuzzy evaluation matrix $\bar{R}^{t_\zeta}=\left(\bar{r}_{ij}^{t_\zeta}\right)_{n\times m}$ is constructed using Eq. (7). At the same time, the product feature weights for different discrete time sequences are calculated, and according to Definition 3, the feature-weighted sentimental intuitionistic fuzzy evaluation matrix $R^{t_\zeta}=\left(<\mu_{ij}^{t_\zeta},\upsilon_{ij}^{t_\zeta}>\right)_{n\times m}$ for different time sequences is calculated using Eq. (8).

**Step 5** The discrete time sequence weight $\eta(t_\zeta)$ is calculated using Eq. (10). The discrete time sequence weighted synthesis evaluation matrix is calculated using the DDIFWAA operator and the DDIFWGAA operator, and the discrete dynamic sentiment intuitionistic fuzzy synthesis decision matrix $R=\left(<\mu_{ij},\upsilon_{ij}>\right)_{n\times m}$ is obtained using Eq. (13) or Eq. (14).

**Step 6** Using Definition 2, the intuitionistic fuzzy score $S(a)$ of each product feature is derived. By comparing the intuitionistic fuzzy scores $S(a)$ of different product features, the rankings of the product features are presented.

**Step 7** The "vertical projection" distances between each solution and the positive or negative ideal solution $V_i^+$ or $V_i^-$ are calculated using Eq. (20) and Eq. (21), and the comprehensive product rankings are presented.

**TABLE 5.** Product feature dictionaries.

| Feature | Word phrases |
|---|---|
| Quality | Quality, signal, character, workmanship, crack, scratch, art…20 in total |
| Appearance | Screen, sense of touch, appearance, effect, package, impression, color, black… 88 in total |
| System | Speed, iOS, system, battery, function, software, duration……31 in total |
| Configuration | Camera, accessory, CPU, capacity, Internet speed, hardware, audio, button…75 in total |
| Price | Price, cost, cost performance, discount…13 in total |
| Brand | Mi, iPhone, Meizu, brand, flagship, redmi, China, model … 41 in total |

## VI. CASE STUDY

Five mobile phones, i.e., iPhone 8, Mate 10, Meizu Pro7, Mi6 and VIVO X20, are analyzed in this case study. We let $A^{t_\zeta}=\left\{A_1^{t_\zeta},A_2^{t_\zeta},A_3^{t_\zeta},A_4^{t_\zeta},A_5^{t_\zeta}\right\}$ denote the alternative set for the previously described phones in time sequence $t_\zeta=a,f,o,\cdots,z\ (\zeta=1,2,\cdots,k)$ . The product features considered are quality, appearance, system, configuration, price and brand. The feature sets associated with each time sequence are represented by $C^{t_\zeta}=\left\{C_1^{t_\zeta},C_2^{t_\zeta},C_3^{t_\zeta},C_4^{t_\zeta},C_5^{t_\zeta},C_6^{t_\zeta}\right\}$.

According to step 1, the feature and sentiment dictionaries based on Python are listed in Tables 5 and 6.

**TABLE 6.** Sentiment dictionaries.

| Orientation | Sentiment word phrases |
|---|---|
| Positive | Smooth, excellent, marvelous, shining, novel, elegant, favorite, fast … 58 in total |
| Neutral | Ok, not bad, normal, ordinary, common…11 in total |
| Negative | Fake, old, malfunction, defect, bad review, not good, very bad, disaster … 36 in total |

According to step 2, we determine each discrete time sequence. Reviews for the 5 smart phones over 3 months (not consecutive) are extracted as a discrete data source. As such, the difference in impact between the release of the phone and the last day of data extraction (May 5th, 2018) can be analyzed for each phone. The time sequences are shown in Table 7.

**TABLE 7.** Discrete time sequences for each product.

| Product | Release date | Time to market | Sequence 1 | Sequence 2 | Sequence 3 |
|---|---|---|---|---|---|
| iPhone 8 | 2017.9.13 | 2017.9.15 | 2017.9.16-2017.10.16 | 2017.12.25-2018.1.25 | |
| Mate 10 | 2017.10.16 | 2017.10.20 | 2017.10.21-2017.11.21 | 2018.01.11-2018.02.11 | |
| Meizu PRO7 | 2017.7.26 | 2017.8.5 | 2017.8.6-2017.9.6 | 2017.12.04-2018.1.04 | 2018.4.5-2018.5.5 |
| Mi6 | 2017.4.9 | 2017.4.28 | 2017.4.29-2017.5.29 | 2017.10.16-2017.11.16 | |
| VIVO X20 | 2017.9.21 | 2017.9.30 | 2017.10.1-2017.11.1 | 2018.01.01-2018.02.01 | |

Because the time to market and the release date of a product may not be the same, Sequence 1 is taken from the next 31 days starting at the time to market. Sequence 2 is taken from the previous 31 days starting at the middle date between the time to market and April 5th. Sequence 3 is taken from April 5th to May 5th (the last 31 days of data collection). We utilize the relative weight method to process the time weight, which does not require the time sequence from a single month but rather focuses on the relative changes in the weights. Therefore, the time sequence numbers are denoted 1, 3 and 5.

According to step 3, the weighted sentiment orientation matrix of reviews from different discrete time sequences is obtained, as shown in Table 8.

**TABLE 8.** Weighted sentiment orientation matrix.

| Alternative product | Quality | | | Appearance | | | System | | |
|---|---|---|---|---|---|---|---|---|---|
| | positive | neutral | negative | positive | neutral | negative | positive | neutral | Negative |
| Time 1 | | | | | | | | | |
| iPhone 8 | 106 | 52.7 | 36.13 | 647.1 | 207 | 83.1 | 452 | 75.01 | 62.5 |
| Mate 10 | 393 | 145 | 25.98 | 2159 | 608 | 125 | 1872 | 272.1 | 194 |
| Meizu PRO7 | 42.4 | 5.24 | 3.514 | 305.7 | 97.2 | 17.5 | 126 | 33.63 | 15.8 |
| Mi6 | 11.9 | 1.46 | 4.47 | 89.03 | 28.3 | 10.5 | 57.8 | 14.88 | 10.4 |
| VIVO X20 | 123 | 40.3 | 11.55 | 1363 | 344 | 59.8 | 703 | 104.2 | 67.5 |
| Time 2 | | | | | | | | | |
| iPhone 8 | 277 | 77.6 | 19.81 | 488.4 | 162 | 68.8 | 570 | 77.58 | 91.2 |
| Mate 10 | 378 | 134 | 25.03 | 1821 | 549 | 103 | 1699 | 281.8 | 227 |
| Meizu PRO7 | 74.9 | 44.9 | 6.431 | 681.1 | 248 | 53.3 | 347 | 95.84 | 38.2 |
| Mi6 | 111 | 45.8 | 7.861 | 435.1 | 200 | 37.7 | 393 | 94.73 | 37.8 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| VIVO X20 | 358 | 126 | 24.76 | 3855 | 872 | 130 | 1860 | 276.6 | 193 |
| Time 3 | | | | | | | | | |
| iPhone 8 | 614 | 178 | 55.43 | 2644 | 791 | 259 | 2431 | 359.5 | 309 |
| Mate 10 | 333 | 100 | 21.08 | 1910 | 592 | 169 | 1606 | 234.5 | 272 |
| Meizu PRO7 | 127 | 62.9 | 21.15 | 1038 | 411 | 89.4 | 556 | 142.3 | 83.8 |
| Mi6 | 205 | 120 | 50 | 1207 | 480 | 177 | 792 | 165.7 | 187 |
| VIVO X20 | 163 | 79.2 | 15.87 | 1312 | 513 | 65.8 | 670 | 150.9 | 88.9 |

Table 8 continued

| Alternative product | Configuration | | | Price | | | Brand | | |
|---|---|---|---|---|---|---|---|---|---|
| | positive | neutral | negative | positive | neutral | negative | positive | neutral | negative |
| Time 1 | | | | | | | | | |
| iPhone 8 | 333.9 | 114.9 | 90.8 | 77.52 | 22.73 | 26.3 | 996.1 | 457.2 | 164 |
| Mate 10 | 1110 | 313.2 | 91.7 | 186.4 | 63.54 | 63.3 | 2957 | 1088 | 225 |
| Meizu PRO7 | 148.4 | 56.52 | 15.8 | 14.16 | 7.093 | 0 | 392.7 | 124 | 26.4 |
| Mi6 | 69.73 | 19.27 | 11.9 | 5.977 | 1.498 | 1.49 | 187.1 | 78.65 | 20.7 |
| VIVO X20 | 932.4 | 191 | 75 | 67.24 | 5.727 | 11.5 | 2340 | 782.3 | 120 |
| Time 2 | | | | | | | | | |
| iPhone 8 | 203.9 | 86.22 | 75.6 | 148.4 | 46.48 | 11.1 | 1197 | 416.3 | 179 |
| Mate 10 | 947.1 | 294.3 | 122 | 141.3 | 40.98 | 18.1 | 3159 | 1130 | 263 |
| Meizu PRO7 | 310.4 | 98.45 | 91.9 | 77.06 | 32.01 | 10.7 | 1083 | 466.6 | 126 |
| Mi6 | 295.3 | 115.2 | 74.8 | 111.2 | 53.51 | 19.8 | 1002 | 534.7 | 93.1 |
| VIVO X20 | 2010 | 455.2 | 225 | 200 | 65.1 | 20.2 | 5459 | 1890 | 291 |
| Time 3 | | | | | | | | | |
| iPhone 8 | 962.4 | 277.7 | 158 | 757.8 | 243.3 | 79.1 | 4223 | 1476 | 443 |
| Mate 10 | 943 | 273.6 | 180 | 166.4 | 45.01 | 63.3 | 2910 | 1109 | 331 |
| Meizu PRO7 | 423.8 | 208.2 | 126 | 140 | 52.99 | 21 | 1690 | 685.9 | 208 |
| Mi6 | 974.2 | 372.2 | 273 | 202.2 | 89.36 | 29 | 2449 | 1179 | 450 |
| VIVO X20 | 717.6 | 200.3 | 121 | 68.38 | 28.97 | 7.86 | 1829 | 907.4 | 123 |

According to step 4 and Table 8, the sentiment intuitionistic fuzzy decision matrix is derived, and the feature weights for different time sequences are obtained. The results are shown in Table 9.

**TABLE 9. Feature weights for different time sequences.**

| | Quality | Appearance | System | Configuration | Price | Brand |
|---|---|---|---|---|---|---|
| Time sequence 1 | 0.145 | 0.205 | 0.189 | 0.165 | 0.130 | 0.166 |
| Time sequence 2 | 0.168 | 0.191 | 0.195 | 0.133 | 0.157 | 0.156 |
| Time sequence 3 | 0.159 | 0.192 | 0.186 | 0.147 | 0.154 | 0.161 |

According to step 5, we can calculate the weights of the three time sequences $\eta(t_\zeta)(t_1=1, t_2=3, t_3=5)$, and λ=0.5. When λ=1, the decay rate is the highest; when λ=0, the decay rate is the lowest. We assume an average decay rate; thus, λ=0.5 is selected. Therefore, the weights are:

$$\begin{cases} \eta(t_1)=\eta(1)=0.0090 \\ \eta(t_2)=\eta(3)=0.2447 \\ \eta(t_3)=\eta(5)=0.6652 \end{cases}$$

Based on the DDIFWGAA operator, the discrete dynamic intuitionistic fuzzy comprehensive decision matrix is shown in Table 11.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access



**TABLE 10. Table 10 Sentiment intuitionistic fuzzy review matrix weighted by features**

| Alternative product | Quality | | Appearance | | System | | Configuration | | Price | | Brand | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | positive | negative | positive | negative | positive | negative | positive | negative | positive | negative | positive | negative |
| Time 1 | | | | | | | | | | | | |
| iPhone 8 | 0.107 | 0.783 | 0.213 | 0.609 | 0.241 | 0.654 | 0.147 | 0.745 | 0.116 | 0.816 | 0.147 | 0.684 |
| Mate 10 | 0.159 | 0.640 | 0.245 | 0.526 | 0.263 | 0.624 | 0.196 | 0.630 | 0.111 | 0.813 | 0.178 | 0.613 |
| Meizu PRO7 | 0.226 | 0.678 | 0.233 | 0.522 | 0.213 | 0.634 | 0.168 | 0.647 | 0.133 | 0.224 | 0.192 | 0.605 |
| Mi6 | 0.148 | 0.818 | 0.217 | 0.599 | 0.202 | 0.675 | 0.176 | 0.703 | 0.133 | 0.792 | 0.161 | 0.646 |
| VIVO X20 | 0.162 | 0.674 | 0.261 | 0.500 | 0.265 | 0.616 | 0.220 | 0.633 | 0.186 | 0.772 | 0.192 | 0.578 |
| Time 2 | | | | | | | | | | | | |
| iPhone 8 | 0.202 | 0.611 | 0.195 | 0.639 | 0.251 | 0.664 | 0.103 | 0.810 | 0.182 | 0.632 | 0.158 | 0.699 |
| Mate 10 | 0.184 | 0.598 | 0.225 | 0.545 | 0.249 | 0.641 | 0.146 | 0.725 | 0.175 | 0.686 | 0.168 | 0.642 |
| Meizu PRO7 | 0.140 | 0.607 | 0.202 | 0.573 | 0.221 | 0.610 | 0.121 | 0.798 | 0.150 | 0.684 | 0.149 | 0.668 |
| Mi6 | 0.171 | 0.601 | 0.180 | 0.577 | 0.236 | 0.598 | 0.118 | 0.779 | 0.135 | 0.704 | 0.138 | 0.640 |
| VIVO X20 | 0.184 | 0.603 | 0.260 | 0.501 | 0.269 | 0.615 | 0.168 | 0.718 | 0.173 | 0.660 | 0.177 | 0.601 |
| Time 3 | | | | | | | | | | | | |
| iPhone 8 | 0.186 | 0.648 | 0.215 | 0.600 | 0.248 | 0.651 | 0.158 | 0.726 | 0.170 | 0.669 | 0.171 | 0.654 |
| Mate 10 | 0.189 | 0.614 | 0.215 | 0.588 | 0.233 | 0.683 | 0.153 | 0.740 | 0.134 | 0.798 | 0.163 | 0.660 |
| Meizu PRO7 | 0.136 | 0.694 | 0.194 | 0.578 | 0.206 | 0.660 | 0.113 | 0.768 | 0.151 | 0.700 | 0.157 | 0.666 |
| Mi6 | 0.118 | 0.726 | 0.182 | 0.635 | 0.197 | 0.714 | 0.127 | 0.770 | 0.142 | 0.691 | 0.138 | 0.701 |
| VIVO X20 | 0.147 | 0.642 | 0.204 | 0.524 | 0.220 | 0.649 | 0.159 | 0.729 | 0.149 | 0.671 | 0.152 | 0.602 |

**TABLE 11. Discrete dynamic intuitionistic fuzzy comprehensive decision matrix**

| Alternative product | Quality | | Appearance | | System | | Configuration | | Price | | Brand | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | positive | negative | positive | negative | positive | negative | positive | negative | positive | negative | positive | negative |
| iPhone 8 | 0.180 | 0.655 | 0.210 | 0.610 | 0.248 | 0.655 | 0.141 | 0.751 | 0.167 | 0.677 | 0.166 | 0.668 |
| Mate 10 | 0.185 | 0.612 | 0.220 | 0.573 | 0.240 | 0.668 | 0.154 | 0.727 | 0.140 | 0.776 | 0.166 | 0.651 |
| Meizu PRO7 | 0.143 | 0.673 | 0.199 | 0.572 | 0.210 | 0.646 | 0.119 | 0.767 | 0.149 | 0.661 | 0.158 | 0.661 |
| Mi6 | 0.132 | 0.710 | 0.184 | 0.619 | 0.206 | 0.685 | 0.128 | 0.767 | 0.140 | 0.705 | 0.140 | 0.682 |
| VIVO X20 | 0.157 | 0.636 | 0.221 | 0.516 | 0.235 | 0.638 | 0.166 | 0.719 | 0.158 | 0.679 | 0.161 | 0.600 |

**TABLE 12. Intuitionistic fuzzy scores of each feature, S(a)**

| Alternative product | Quality | | Appearance | | System | | Configuration | | Price | | Brand | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | S(a) | ranking | S(a) | ranking | S(a) | ranking | S(a) | ranking | S(a) | ranking | S(a) | ranking |
| iPhone 8 | -0.474 | 2 | -0.401 | 4 | -0.407 | 2 | -0.61 | 3 | -0.511 | 1 | -0.503 | 3 |
| Mate 10 | -0.427 | 1 | -0.353 | 2 | -0.428 | 3 | -0.573 | 2 | -0.636 | 5 | -0.486 | 2 |
| Meizu PRO7 | -0.53 | 4 | -0.373 | 3 | -0.436 | 4 | -0.648 | 5 | -0.512 | 2 | -0.503 | 4 |
| Mi6 | -0.578 | 5 | -0.434 | 5 | -0.479 | 5 | -0.639 | 4 | -0.565 | 4 | -0.542 | 5 |
| VIVO X20 | -0.479 | 3 | -0.295 | 1 | -0.403 | 1 | -0.553 | 1 | -0.521 | 3 | -0.439 | 1 |

According to step 5, the intuitionistic fuzzy score of each feature can be calculated from the score function. The results are shown in Table 12.

According to the feature scores and rankings in Table 12, VIVO X20 ranks first in terms of appearance, configuration and brand, and iPhone 8 ranks first in terms of system and price, which is consistent with the results obtained using the DDIFWAA operator.

We further investigate the overall rank of each phone. According to step 7, we calculate the vertical distance between each product and the positive/negative ideal product. The results are shown in Table 13.

**TABLE 13. Product ranking based on the DDIFWGAA operator and vertical distance.**

| Alternative product | $V_i^+$ | $V_i^-$ | Ranking |
|---|---|---|---|
| iPhone 8 | 0.3877 | 0.368 | 2 |
| Mate 10 | 0.4631 | 0.2926 | 3 |
| Meizu PRO7 | 0.5208 | 0.2349 | 4 |
| Mi6 | 2.1952 | -1.439 | 5 |
| VIVO X20 | -1.75 | 2.5061 | 1 |

The overall ranking is VIVO X20>iPhone 8>Mate10>Meizu PRO7>Mi6. Therefore, VIVO X20 is the best choice for customers. We conclude that the overall ranking is in alignment with the feature ranking. Moreover, the DDIFWAA operator leads to the same overall ranking for alternative phones.

## VII. DISCUSSION

### A. COMPARISION ANALYSIS

To show the advantages of our proposed method, we compared with the existing main methods. Of note, although many existing methods have provided ideas for solving product ranking and online purchase decision problems, these methods are different from the method in this paper. The main differences lie in the time-series weight and dynamic review information aggregation. The problems considered in the existing methods are mainly objective product rankings based on a continuous time series [2-9, 11-13]. That is, the discrete distribution features of the actual review information records and the consumer behavior of discrete random reading review information are neglected. For example, the methods of [4-9, 11-12] do not consider the time-series weight and completely ignore the influence of review information in different time series on consumers' purchase decisions. In the methods of [2-3, 13], although the different time-series weights are considered, only equal weights of continuous time series are considered, which ignores the different influences of different time-series review information in the process of purchase decisions. Moreover, it is difficult for the time-series weight calculation methods of [2-3, 13] to describe the difference degree in consumer subjective preference for different time-series review information. Furthermore, the information aggregation method in the existing studies cannot aggregate discrete random dynamic information. In the actual online shopping process, consumers are more reliant on the recent review information and less reliant on the long-dated review information; thus, the previously described methods violated the principle of "stressing the present rather than the past". In contrast, the problem considered in this proposed method involves purchase decisions based on consumers' preferences for different time-series review information. This dynamic information preference is completely based on consumer online shopping behavior habit and discrete distribution feature of review information. The weight calculation model of this paper can significantly widen the gap between different discrete time-series weights and gives more weight to recent information. Compared with the traditional DIFWA operator, the DDIFWA operator in this paper can increase the degree of recent information extraction, thus following the principle of "stressing the present rather than the past".

**TABLE 14.** **The alternatives ranking without time preference.**

| Method | Ranking result |
|---|---|
| the proposed method | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |
| the method in [2] | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |
| the method in [3] | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |
| the method in [6] | VIVO X20> Meizu PRO7> Mate10> Mi6> iPhone 8 |
| the method in [8] | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |
| the method in [9] | VIVO X20> Mate10> Meizu PRO7> iPhone 8> Mi6 |
| the method in [13] | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |

**TABLE 15.** **The alternatives ranking with time preference**

| Discrete time weights | | | Ranking result |
|---|---|---|---|
| $\eta(t_1)$ | $\eta(t_2)$ | $\eta(t_3)$ | |
| 1.000 | 0.000 | 0.000 | Meizu PRO7>VIVO X20> Mate10> Mi6 > iPhone 8 |
| 0.650 | 0.250 | 0.100 | Meizu PRO7>VIVO X20> Mate10> Mi6 > iPhone 8 |
| 0.333 | 0.333 | 0.333 | VIVO X20> Meizu PRO7> Mate10> iPhone 8> Mi6 |
| 0.100 | 0.250 | 0.650 | VIVO X20 > Mate 10> iPhone 8> Meizu PRO7> Mi6 |
| 0.000 | 0.000 | 1.000 | VIVO X20 > iPhone 8> Mate 10> Meizu PRO7> Mi6 |

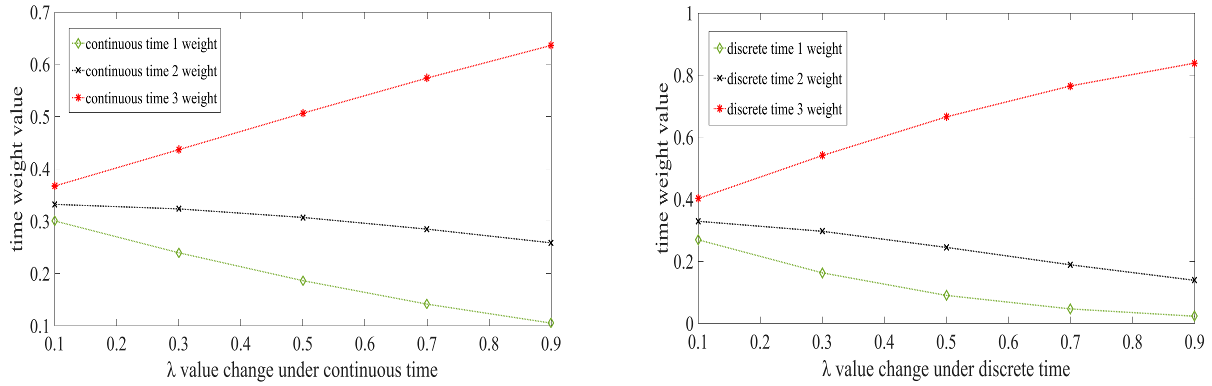
Furthermore, in general, different methods can be used for comparison when solving the same problem [2-3]. To this end, we compare the methods without a time-series information preference; that is, the weights of different time series are equal or not considered. The result based on the proposed method is approximately the same as that in the methods [2-3, 6, 8-9, 13], and the comparison results are shown in Table 14. It is necessary to further illustrate that the methods proposed in the literature [4-5, 7, 11-12] are not used because additional information or constraint conditions are needed in these studies. However, if the discrete time weights are considered, existing methods cannot be used to obtain the ranking results of alternative products because discrete time weight weights based on consumers' subjective preferences are not considered in the existing studies [2-9, 11-13]. Table 15 shows the ranking results of the alternative products obtained by the proposed method, where different discrete time weights are considered. As indicated in Table 15, using the proposed method, different ranking results could be obtained if

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access


different discrete time weights are considered. It means that using the proposed method, different products could be recommended to the consumers with different subjective preferences on discrete time weights. When consumers value recent information more, the best choice is VIVO X20. In contrast, if consumers value the farthest historical review information at the beginning of the mobile phone release, the best choice is Meizu PRO7.

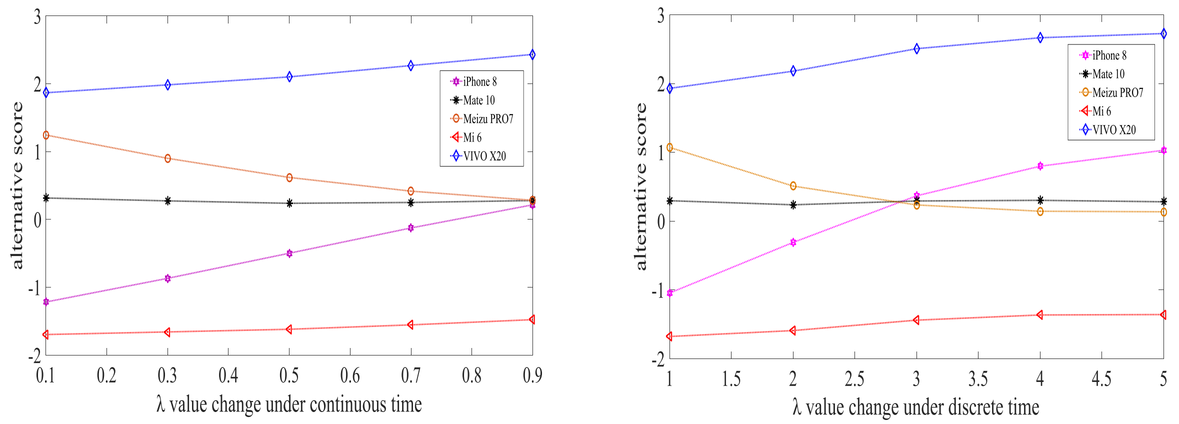
### B. SENSITIVITY ANALYSIS

The parameter $\lambda$ in Eq. (2) can affect the results of time weights and product rankings, and the effects of a continuous time sequence and a discrete time sequence are different. To compare the differences, a sensitivity analysis is conducted to evaluate the impact of $\lambda$ on the time weight and product ranking. $\lambda$ varies over the range from 0.1 to 0.9, and the results are shown in Figs. 1 and 2.




(a) (b)

**FIGURE 1. The impact of $\lambda$ on time weights.**




(a) (b)

**FIGURE 2. The impact of $\lambda$ on product ranking.**

As shown in Fig. 1, as $\lambda$ approaches 1 (that is, as consumers pay more attention to the most recent information), the difference between the time weights is larger for a discrete time series than for a continuous time series. Furthermore, the time 1 weight value, which represents the most recent time, is larger for a discrete time sequence than for a continuous time series when $\lambda$ is fixed. Fig. 2(a) shows that $\lambda$ has little influence on the product ranking for a continuous time series and that the ranking of the five products is nearly fixed at VIVO X20>Meizu PRO 7>Mate 10>iPhone 8 >Mi 6. In contrast, for a discrete time series, as shown in Fig. 2(b), the product ranking is more sensitive to $\lambda$. It is clear that the product ranking as $\lambda$ is becoming smaller is quite different from the product ranking as $\lambda$ is becoming bigger. As indicated, the discrete time series has a significant impact on the product ranking. Therefore, compared with continuous time-series results, discrete time-series results are more meaningful.

Furthermore, Baird noted that a sensitivity analysis determines how potential changes in inputs can affect calculation error values and rankings [72]. In this paper, a sentiment IFN is derived using semantic analysis. Due to the diversity of the language, error in the calculated number of emotion words is inevitable. Therefore, this paper considers the existence of calculation error and analyzes the influence of changes in the emotional orientation value of each product feature on the product ranking.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2921403, IEEE Access



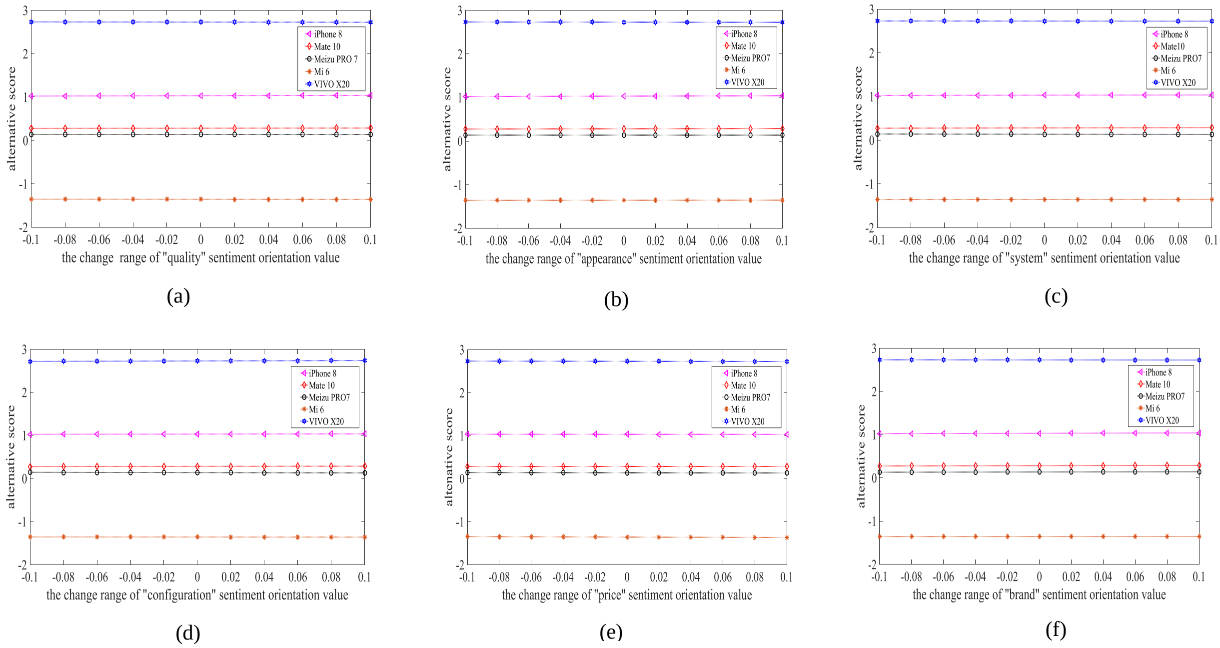



**FIGURE 3. The impact of a change in feature emotional orientation value on product ranking.**

As shown in Fig. 3, the impact of a change in the 6 feature emotional orientation values on the product ranking is successively evaluated. Considering the positive emotional orientation value of each feature as an example, the error range of the emotional orientation value is set to $[-10\%, 10\%]$ with the same step size of $0.02$. The results are shown in Fig. 3(a)–(f). As shown, the product ranking is unchanged, which indicates that the proposed model is robust against errors to some extent.

## VIII. CONCLUSION

This paper presents a novel method based on DDIFWA operators for decision-level sentiment analysis. The method consists of three parts: (1) Data acquisition and processing. Online reviews on five selected mobile phone products were obtained by the web crawler technology from JD Mall and Tmall Mall, and the open-source Chinese word segmentation package jieba is used for word segmentation and POS tagging to extract product feature words (nouns) and corresponding sentiment words (adjectives). (2) Identification of sentiment orientations of the online reviews based on building a dictionary of emotions. The online reviews of the alternative products concerning multiple attributes are preprocessed, and feature and sentiment dictionaries were built to classify the sentiment orientations of online reviews into three categories: positive, neutral, and negative. (3) Product feature evaluation and purchase decision method based on sentiment intuitionistic fuzzy numbers. Sentiment intuitionistic fuzzy evaluation matrixes are constructed at different discrete times, and two DDIFWA operators are presented and used to fuse positive, neutral and negative sentiment orientation information of evaluation matrixes. Furthermore, the intuitionistic fuzzy score and "vertical projection" distance method are employed to determine rankings of the alternative products and their features. A case study is provided to illustrate the use of the proposed method, and comparisons and sensitivity analysis are further conducted to illustrate the characteristics and advantages of the proposed method.

Our method has several limitations, including those related to the dynamics of language evolution and integration with other types of fuzzy sets; thus, in the near future, we will investigate: 1) classification of sentiment words in greater detail to increase the diversification of the results; 2) the use of the web crawler to capture more accurate reviews by considering fake reviews at the feature and sentiment levels and 3) integration with other types of fuzzy sets, such as hesitant fuzzy sets and Pythagorean fuzzy sets, to identify more features of consumption behavior and purchase decision making.

## APPENDIX

**Theorem** 1. Let $a(t)=(a(t_1),a(t_2),a(t_3),\cdots,a(t_k))$ $=(a(a),a(f),a(o),\cdots,a(z))$ be the IFNs of the $k$ time sequences and $\eta(t_\varsigma)=(\eta(t_1),\eta(t_2),\eta(t_3),\cdots,\eta(t_k))^T$ $=(\eta(a),\eta(f),\eta(o),\cdots,\eta(z))^T$ represent the discrete time weight vector, $\eta(t_\varsigma)\in[0,1]$, $\sum_{t_\varsigma=t_1=a}^{t_k=z}\eta(t_\varsigma)=1$. Let $DDIFWAA: Q^n \to Q$, then:

$$DDIFWAA_{\eta(t_\varsigma)}(a(a),a(f),a(o),\cdots,a(z))$$
$$=\sum_{t_\varsigma=t_1=a}^{t_k=z}\eta(t_\varsigma)\times a(t_\varsigma)=(1-\prod_{t_\varsigma=t_1=a}^{t_k=z}(1-\mu_{(t_\varsigma)})^{\eta(t_\varsigma)},\prod_{t_\varsigma=t_1=a}^{t_k=z}\upsilon_{(t_\varsigma)}{}^{\eta(t_\varsigma)})$$

is a DDIFWAA operator.

**Proof:**

1) If $\zeta=2$, i.e., $t_\zeta=a,f\ (\zeta=1,2)$

$a(a)\times\eta(a)=(1-(1-\mu_{(a)})^{\eta(a)},\upsilon_{(a)}{}^{\eta(a)})$ ,
$a(f)\times\eta(f)=(1-(1-\mu_{(f)})^{\eta(f)},\upsilon_{(f)}{}^{\eta(f)})$

then:

$$\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a),a(f))=a(a)\times\eta(a)\oplus a(f)\times\eta(f)\\
&=(1-(1-\mu_{(a)})^{\eta(a)},\upsilon_{(a)}{}^{\eta(a)})\oplus(1-(1-\mu_{(f)})^{\eta(f)},\upsilon_{(f)}{}^{\eta(f)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}+1-(1-\mu_{(f)})^{\eta(f)}-(1-(1-\mu_{(a)})^{\eta(a)})\\
&\quad\cdot(1-(1-\mu_{(f)})^{\eta(f)}),\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}\cdot(1-\mu_{(f)})^{\eta(f)},\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)})
\end{aligned}$$

If $\zeta=3$, i.e., $t_\zeta=a,f,o\,(\zeta=1,2,3)$

then:

$$\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a),a(f),a(o))=a(a)\times\eta(a)\\
&\quad\oplus a(f)\times\eta(f)\oplus a(o)\times\eta(o)\\
&=(1-(1-\mu_{(a)})^{\eta(a)},\upsilon_{(a)}{}^{\eta(a)})\oplus(1-(1-\mu_{(f)})^{\eta(f)},\\
&\quad\upsilon_{(f)}{}^{\eta(f)})\oplus(1-(1-\mu_{(o)})^{\eta(o)},\upsilon_{(o)}{}^{\eta(o)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}+1-(1-\mu_{(f)})^{\eta(f)}-(1-(1-\mu_{(a)})^{\eta(a)})\\
&\quad\cdot(1-(1-\mu_{(f)})^{\eta(f)}),\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)})\\
&\quad\oplus(1-(1-\mu_{(o)})^{\eta(o)},\upsilon_{(o)}{}^{\eta(o)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}+1-(1-\mu_{(f)})^{\eta(f)}-(1-(1-\mu_{(a)})^{\eta(a)})\\
&\quad\cdot(1-(1-\mu_{(f)})^{\eta(f)})+1-(1-\mu_{(o)})^{\eta(o)}\\
&\quad-((1-(1-\mu_{(a)})^{\eta(a)}+1-(1-\mu_{(f)})^{\eta(f)}\\
&\quad-(1-(1-\mu_{(a)})^{\eta(a)})\\
&\quad\cdot(1-(1-\mu_{(f)})^{\eta(f)}))\cdot(1-(1-\mu_{(o)})^{\eta(o)}),\\
&\quad\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)}\cdot\upsilon_{(o)}{}^{\eta(o)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}\cdot(1-\mu_{(f)})^{\eta(f)}\\
&\quad\cdot(1-\mu_{(o)})^{\eta(o)},\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)}\cdot\upsilon_{(o)}{}^{\eta(o)})
\end{aligned}$$

2) If $\zeta=k,(k>3)$

$$\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a),a(f),a(o),\cdots,a(z))=a(a)\times\eta(a)\\
&\quad\oplus a(f)\times\eta(f)\oplus a(o)\times\eta(o)\oplus\cdots\oplus a(z)\times\eta(z)\\
&=(1-(1-\mu_{(a)})^{\eta(a)},\upsilon_{(a)}{}^{\eta(a)})\oplus(1-(1-\mu_{(f)})^{\eta(f)},\upsilon_{(f)}{}^{\eta(f)})\\
&\quad\oplus(1-(1-\mu_{(o)})^{\eta(o)},\upsilon_{(o)}{}^{\eta(o)})\oplus\cdots\\
&\quad\oplus(1-(1-\mu_{(z)})^{\eta(z)},\upsilon_{(z)}{}^{\eta(z)})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}\cdot(1-\mu_{(f)})^{\eta(f)}\cdot(1-\mu_{(o)})^{\eta(o)}\\
&\quad\cdots\cdots(1-\mu_{(z)})^{\eta(z)},\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)}\cdot\upsilon_{(o)}{}^{\eta(o)}\cdots\cdots\upsilon_{(z)}{}^{\eta(z)})\\
&=(1-\prod_{\zeta=1}^{k}(1-\mu_{(t_\zeta)})^{\eta(t_\zeta)},\prod_{\zeta=1}^{k}\upsilon_{(t_\zeta)}{}^{\eta(t_\zeta)})
\end{aligned}$$

3) If $\zeta=k+1(t_{k+1}=z')$

$$\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a),a(f),a(o),\cdots,a(z),a(z'))\\
&=a(a)\times\eta(a)\oplus a(f)\times\eta(f)\oplus\cdots\\
&\quad\oplus a(z)\times\eta(z)\oplus a(z')\times\eta(z')\\
&=(1-(1-\mu_{(a)})^{\eta(a)},\upsilon_{(a)}{}^{\eta(a)})\oplus(1-(1-\mu_{(f)})^{\eta(f)}\\
&\quad,\upsilon_{(f)}{}^{\eta(f)})\oplus\cdots\oplus(1-(1-\mu_{(z)})^{\eta(z)},\upsilon_{(z)}{}^{\eta(z)})\\
&\quad\oplus(1-(1-\mu_{(z')})^{\eta(z')},\upsilon_{(z')}{}^{\eta(z')})\\
&=(1-(1-\mu_{(a)})^{\eta(a)}\cdot(1-\mu_{(f)})^{\eta(f)}\cdots\cdots(1-\mu_{(z)})^{\eta(z)}\\
&\quad\cdot(1-\mu_{(z')})^{\eta(z')},\upsilon_{(a)}{}^{\eta(a)}\cdot\upsilon_{(f)}{}^{\eta(f)}\cdots\cdots\upsilon_{(z)}{}^{\eta(z)}\cdot\upsilon_{(z')}{}^{\eta(z')})\\
&=(1-\prod_{\zeta=1}^{k}(1-\mu_{(t_\zeta)})^{\eta(t_\zeta)}\cdot(1-\mu_{(z')})^{\eta(z')}\\
&\quad,\prod_{\zeta=1}^{k}\upsilon_{(t_\zeta)}{}^{\eta(t_\zeta)}\cdot\upsilon_{(z')}{}^{\eta(z')})\\
&=(1-\prod_{\zeta=1}^{k+1}(1-\mu_{(t_\zeta)})^{\eta(t_\zeta)},\prod_{\zeta=1}^{k+1}\upsilon_{(t_\zeta)}{}^{\eta(t_\zeta)})
\end{aligned}$$

If $\zeta=k+1$, Theorem 1 holds.

This completes the proof of Theorem 1.

**Theorem 2.** Let $a(t)=(a(t_1),a(t_2),a(t_3),\cdots,a(t_k))$ $=(a(a),a(f),a(o),\cdots,a(z))$ be the IFNs of the $k$ time sequences and $\eta(t_\zeta)=(\eta(t_1),\eta(t_2),\eta(t_3),\cdots,\eta(t_k))^T$ $=(\eta(a),\eta(f),\eta(o),\cdots,\eta(z))^T$ represent the discrete time weight vector, $\eta(t_\zeta)\in[0,1]$ , $\sum_{t_\zeta=t_1=a}^{t_k=z}\eta(t_\zeta)=1$ . Let $DDIFWGAA:Q^n\to Q$ , then:

$$\begin{aligned}
&DDIFWGAA_{\eta(t_\zeta)}(a(a),a(f),a(o),\cdots,a(z))\\
&=\prod_{t_\zeta=t_1=a}^{t_k=z}\eta(t_\zeta)\times a(t_\zeta)=(\prod_{t_\zeta=t_1=a}^{t_k=z}\mu_{(t_\zeta)}{}^{\eta(t_\zeta)},1-\prod_{t_\zeta=t_1=a}^{t_k=z}(1-\upsilon_{(t_\zeta)})^{\eta(t_\zeta)})
\end{aligned}$$

is a DDIFWGAA operator.

**Proof:**

1) If $\zeta=2$, i.e., $t_\zeta=a,f\ (\zeta=1,2)$

$a(a)^{\eta(a)} = (\mu_{(a)}{}^{\eta(a)}, 1-(1-\upsilon_{(a)})^{\eta(a)})$ , $a(f)^{\eta(f)} = (\mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(f)})^{\eta(f)})$

then:

$$
\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a), a(f)) = a(a)^{\eta(a)} \otimes a(f)^{\eta(f)} \\
&= (\mu_{(a)}{}^{\eta(a)}, 1-(1-\upsilon_{(a)})^{\eta(a)}) \otimes (\mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(f)})^{\eta(f)}) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(a)})^{\eta(a)} + 1-(1-\upsilon_{(f)})^{\eta(f)} \\
&-(1-(1-\upsilon_{(a)})^{\eta(a)}) \cdot (1-(1-\upsilon_{(f)})^{\eta(f)})) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(a)})^{\eta(a)} \cdot (1-\upsilon_{(f)})^{\eta(f)})
\end{aligned}
$$

If $\zeta = 3$, i.e., $t_\zeta = a, f, o (\zeta = 1,2,3)$

then:

$$
\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a), a(f), a(o)) \\
&= a(a)^{\eta(a)} \otimes a(f)^{\eta(f)} \otimes a(o)^{\eta(o)} \\
&= (\mu_{(a)}{}^{\eta(a)}, 1-(1-\upsilon_{(a)})^{\eta(a)}) \otimes (\mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(f)})^{\eta(f)}) \\
&\otimes (\mu_{(o)}{}^{\eta(o)}, 1-(1-\upsilon_{(o)})^{\eta(o)}) \\
&= (1-(1-\mu_{(a)})^{\eta(a)} + 1-(1-\mu_{(f)})^{\eta(f)} - (1-(1-\mu_{(a)})^{\eta(a)}) \\
&\cdot (1-(1-\mu_{(f)})^{\eta(f)}), \upsilon_{(a)}{}^{\eta(a)} \cdot \upsilon_{(f)}{}^{\eta(f)}) \\
&\otimes (1-(1-\mu_{(o)})^{\eta(o)}, \upsilon_{(o)}{}^{\eta(o)}) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)} \cdot \mu_{(o)}{}^{\eta(o)}, \\
&1-(1-\upsilon_{(a)})^{\eta(a)} + 1-(1-\upsilon_{(f)})^{\eta(f)} - (1-(1-\upsilon_{(a)})^{\eta(a)}) \\
&\cdot (1-(1-\upsilon_{(f)})^{\eta(f)}) + 1-(1-\upsilon_{(o)})^{\eta(o)} \\
&-((1-(1-\upsilon_{(a)})^{\eta(a)} + 1-(1-\upsilon_{(f)})^{\eta(f)} - (1-(1-\upsilon_{(a)})^{\eta(a)}) \\
&\cdot (1-(1-\upsilon_{(f)})^{\eta(f)})) \cdot (1-(1-\upsilon_{(o)})^{\eta(o)})) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)} \cdot \mu_{(o)}{}^{\eta(o)}, 1-(1-\upsilon_{(a)})^{\eta(a)} \\
&\cdot (1-\upsilon_{(f)})^{\eta(f)} \cdot (1-\upsilon_{(o)})^{\eta(o)})
\end{aligned}
$$

2) If $\zeta = k, (k > 3)$

$$
\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a), a(f), a(o), \cdots, a(z)) \\
&= a(a)^{\eta(a)} \otimes a(f)^{\eta(f)} \otimes a(o)^{\eta(o)} \otimes \cdots \otimes a(z)^{\eta(z)} \\
&= (\mu_{(a)}{}^{\eta(a)}, 1-(1-\upsilon_{(a)})^{\eta(a)}) \otimes (\mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(f)})^{\eta(f)}) \\
&\otimes (\mu_{(o)}{}^{\eta(o)}, 1-(1-\upsilon_{(o)})^{\eta(o)}) \otimes \cdots \otimes (\mu_{(z)}{}^{\eta(z)}, 1-(1-\upsilon_{(z)})^{\eta(z)}) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)} \cdot \mu_{(o)}{}^{\eta(o)} \cdots\cdots \mu_{(z)}{}^{\eta(z)}, 1-(1-\upsilon_{(a)})^{\eta(a)} \\
&\cdot (1-\upsilon_{(f)})^{\eta(f)} \cdot (1-\upsilon_{(o)})^{\eta(o)} \cdots\cdots (1-\upsilon_{(z)})^{\eta(z)}) \\
&= (\prod_{\zeta=1}^{k} \mu_{(t_\zeta)}{}^{\eta(t_\zeta)}, 1-\prod_{\zeta=1}^{k} (1-\upsilon_{(t_\zeta)})^{\eta(t_\zeta)})
\end{aligned}
$$

3) If $\zeta = k+1 (t_{k+1} = z')$

$$
\begin{aligned}
&DDIFWAA_{\eta(t_\zeta)}(a(a), a(f), a(o), \cdots, a(o), a(z')) \\
&= a(a)^{\eta(a)} \otimes a(f)^{\eta(f)} \otimes \cdots \otimes a(z)^{\eta(z)} \otimes a(z')^{\eta(z')} \\
&= (\mu_{(a)}{}^{\eta(a)}, 1-(1-\upsilon_{(a)})^{\eta(a)}) \otimes (\mu_{(f)}{}^{\eta(f)}, 1-(1-\upsilon_{(f)})^{\eta(f)}) \\
&\otimes \cdots \otimes (\mu_{(z)}{}^{\eta(z)}, 1-(1-\upsilon_{(z)})^{\eta(z)}) \\
&\otimes (\mu_{(z')}{}^{\eta(z')}, 1-(1-\upsilon_{(z')})^{\eta(z')}) \\
&= (\mu_{(a)}{}^{\eta(a)} \cdot \mu_{(f)}{}^{\eta(f)} \cdots\cdots \mu_{(z)}{}^{\eta(z)} \cdot \mu_{(z')}{}^{\eta(z')}, 1-(1-\upsilon_{(a)})^{\eta(a)} \\
&\cdot (1-\upsilon_{(f)})^{\eta(f)} \cdots\cdots (1-\upsilon_{(z)})^{\eta(z)} \cdot (1-\upsilon_{(z')})^{\eta(z')}) \\
&= (\prod_{\zeta=1}^{k} \mu_{(t_\zeta)}{}^{\eta(t_\zeta)} \cdot \mu_{(z')}{}^{\eta(z')}, 1-\prod_{\zeta=1}^{k} (1-\upsilon_{(t_\zeta)})^{\eta(t_\zeta)} \cdot (1-\upsilon_{(z')})^{\eta(z')}) \\
&= (\prod_{\zeta=1}^{k+1} \mu_{(t_\zeta)}{}^{\eta(t_\zeta)}, 1-\prod_{\zeta=1}^{k+1} (1-\upsilon_{(t_\zeta)})^{\eta(t_\zeta)})
\end{aligned}
$$

If $\zeta = k+1$, Theorem 1 holds.

This completes the proof of Theorem 2.

## REFERENCES


[1] N. Jindal and B. Liu, "Analyzing and detecting review spam," in *7th IEEE International Conference on Data Mining*, Omaha, NE, Oct 28-31 2007, in IEEE International Conference on Data Mining, 2007, pp. 547-552.

[2] Y. Liu, J.-W. Bi, and Z.-P. Fan, "A Method for Ranking Products Through Online Reviews Based on Sentiment Classification and Interval-Valued Intuitionistic Fuzzy TOPSIS," *International Journal of Information Technology & Decision Making*, vol. 16, no. 6, pp. 1497-1522, Nov 2017.

[3] Y. Liu, J.-W. Bi, and Z.-P. Fan, "Ranking products through online reviews: A method based on sentiment analysis technique and intuitionistic fuzzy set theory," *Information Fusion*, vol. 36, pp. 149-161, Jul 2017.

[4] A. Ghose and P. G. Ipeirotis, "Estimating the Helpfulness and Economic Impact of Product Reviews: Mining Text and Reviewer Characteristics," *Ieee Transactions on Knowledge and Data Engineering*, vol. 23, no. 10, pp. 1498-1512, Oct 2011.

[5] T. Lappas, M. Crovella and E. Terzi, “Selecting a characteristic set of reviews,” in Proc. 18th ACM SIGKDD Int. Conf. Knowledge Discovery and Data Mining, Beijing, China, 2012, pp. 832–840.

[6] K. Zhang et al., “Mining millions of reviews: A technique to rank products based on importance of reviews,” in: Proceedings of the 13th International Conference on Electronic Commerce, New York, USA, 2011, pp. 1–8.

[7] K. Zhang, R. Narayanan, and A. Choudhary, “Voice of the customers: mining online customer reviews for product feature-based ranking,” in: WOSN’10 Proceedings of the 3rd Conference on Online social networks, Boston, USA, 2010.

[8] Z.-P. Fan, Y. Xi, and Y. Liu, "Supporting consumer's purchase decision: a method for ranking products based on online multi-attribute product ratings," *Soft Computing*, vol. 22, no. 16, pp. 5247-5261, Aug 2018.

[9] Z. Yang and J. Zhu, “Online product purchasing decision through iot platform: based on normal stochastic multi-criteria decision method and vertical projection distance,” *Cluster Computing*, to be published. DOI: https://doi.org/10.1007/s10586-017-1670-1.

[10] G. Li, G. Kou , and Y. Peng, “Dynamic fuzzy multiple criteria decision making for performance evaluation,” *Technological & Economic Development of Economy*, vol. 21, no. 5, pp. 705-719, 2015.

[11] Y. Peng, G. Kou, and J. Li, "A Fuzzy PROMETHEE Approach for Mining Customer Reviews in Chinese," *Arabian Journal for Science and Engineering*, vol. 39, no. 6, pp. 5245-5252, Jun 2014.

[12] C. Kun, K. Gang, S. Jennifer, and C. Yang, “Visualizing market structure through online product reviews: integrate topic modeling,

topsis, and multi-dimensional scaling approaches," Electronic Commerce Research and Applications, vol. 14, pp. 58-74, 2015.

[13] E. Najmi, K. Hashmi, Z. Malik, A. Rezgui, and H. U. Khan, "CAPRA: a comprehensive approach to product ranking using customer reviews," *Computing,* vol. 97, no. 8, pp. 843-867, Aug 2015.

[14] T. J. Brailsford, J. H. W. Penm, and R. D. Terrell, "Selecting the forgetting factor in subset autoregressive modelling," *Journal of Time Series Analysis,* vol. 23, no. 6, pp. 629-649, Nov 2002.

[15] Z. Yang, J. Li, L. Huang, and Y. Shi, "Developing dynamic intuitionistic normal fuzzy aggregation operators for multi-attribute decision-making with time sequence preference," *Expert Systems with Applications,* vol. 82, pp. 344-356, Oct 1 2017.

[16] R. W. Picard, "Affective Computing," MIT Press, 1997.

[17] G. Rao, W. Huang, Z. Feng, and Q. Cong, "LSTM with sentence representations for document-level sentiment classification," *Neurocomputing,* vol. 308, pp. 49-57, Sep 25 2018.

[18] O. Appel, F. Chiclana, J. Carter, and H. Fujita, "A hybrid approach to the sentiment analysis problem at the sentence level," *Knowledge-Based Systems,* vol. 108, pp. 110-124, Sep 15 2016.

[19] Y. Ren, R. Wang, and D. Ji, "A topic-enhanced word embedding for Twitter sentiment classification," *Information Sciences,* vol. 369, pp. 188-198, Nov 10 2016.

[20] Z. Wang, Z. Wu, and F. Hu, "Words Sentiment Polarity Calculation Based on HowNet and PMI," *Computer Engineering,* vol. 38, no. 15, pp. 187-189,193, 2012.

[21] M. Hu and B. Liu, "Mining opinion features in customer reviews," in: National Conference on Artificial Intelligence, San Jose, California, 2004a, pp. 755–760.

[22] M. Hu and B. Liu, "Mining and summarizing customer reviews," in: Tenth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Seattle, Washington, Usa, August, 2004b, pp. 168–177.

[23] X. Fu, G. Liu, Y. Guo, and Z. Wang, "Multi-aspect sentiment analysis for Chinese online social reviews based on topic modeling and HowNet lexicon," *Knowledge-Based Systems,* vol. 37, pp. 186-195, Jan 2013.

[24] P. D. Turney and M. L. Littman, "Unsupervised learning of semantic orientation from a hundred-billion-word corpus," Technical Report ERB-1094, National Research Council Canada, Institute for Information Technology, 2002.

[25] E. Asgarian, M. Kahani, and S. Sharifi, "The Impact of Sentiment Features on the Sentiment Polarity Classification in Persian Reviews," *Cognitive Computation,* vol. 10, no. 1, pp. 117-135, Feb 2018.

[26] H. Yuan, W. Xu, Q. Li, and R. Lau, "Topic sentiment mining for sales performance prediction in e-commerce," *Annals of Operations Research,* vol. 270, no. 1-2, pp. 553-576, Nov 2018.

[27] S. Xiong, K. Wang, D. Ji, and B. Wang, "A short text sentiment-topic model for product reviews," *Neurocomputing,* vol. 297, pp. 94-102, Jul 5 2018.

[28] Y. Fang, H. Tan, and J. Zhang, "Multi-Strategy Sentiment Analysis of Consumer Reviews Based on Semantic Fuzziness," *Ieee Access,* vol. 6, pp. 20625-20631, 2018.

[29] W. Wang, H. Wang, and Y. Song, "Ranking product aspects through sentiment analysis of online reviews," *Journal of Experimental & Theoretical Artificial Intelligence,* vol. 29, no. 2, pp. 227-246, 2017.

[30] A. Y. L. Chong, E. Ch'ng, M. J. Liu, and B. Li, "Predicting consumer product demands via Big Data: the roles of online promotional marketing and online reviews," *International Journal of Production Research,* vol. 55, no. 17, pp. 5142-5156, 2017.

[31] J. Ren, W. Yeoh, M. S. Ee, and A. Popovic, "Online consumer reviews and sales: Examining the chicken-egg relationships," *Journal of the Association for Information Science and Technology,* vol. 69, no. 3, pp. 449-460, Mar 2018.

[32] M. Siering, J. Muntermann, and B. Rajagopalan, "Explaining and predicting online review helpfulness: The role of content and reviewer-related signals," *Decision Support Systems,* vol. 108, pp. 1-12, Apr 2018.

[33] C. Wang, G. Chen, and Q. Wei, "A temporal consistency method for online review ranking," *Knowledge-Based Systems,* vol. 143, pp. 259-270, Mar 1 2018.

[34] K. Zhao, A. C. Stylianou, and Y. Zheng, "Sources and impacts of social influence from online anonymous user reviews," *Information & Management,* vol. 55, no. 1, pp. 16-30, Jan 2018.

[35] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, pp. 338–353, 1965.

[36] X. Peng, "New Multiparametric Similarity Measure and Distance Measure for Interval Neutrosophic Set With IoT Industry Evaluation," *Ieee Access*, vol. 7, pp. 28258-28280, 2019.

[37] X. Peng and J. Dai, "Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function," *Neural Computing & Applications*, vol. 29, no. 10, pp. 939-954, May 2018.

[38] V. Torra, "Hesitant Fuzzy Sets," *International Journal of Intelligent Systems,* vol. 25, no. 6, pp. 529-539, Jun 2010.

[39] X. Peng and W. Li, "Algorithms for Interval-Valued Pythagorean Fuzzy Sets in Emergency Decision Making Based on Multiparametric Similarity Measures and WDBA," *Ieee Access*, vol. 7, pp. 7419-7441, 2019.

[40] Y. Song and G. Li, "A large-scale group decision-making with incomplete multi-granular probabilistic linguistic term sets and its application in sustainable supplier selection," *Journal of the Operational Research Society*, to be published. DOI: https://doi.org/10.1080/01605682.2018.1458017.

[41] G. Li, G. Kou, C. Lin, L. Xu, and Y. Liao, "Multi-attribute decision making with generalized fuzzy numbers," *Journal of the Operational Research Society,* vol. 66, no. 11, pp. 1793-1803, Nov 2015.

[42] S. Nguyen Thi Kim, "A foundation on semigroups of operators defined on the set of triangular fuzzy numbers and its application to fuzzy fractional evolution equations," *Fuzzy Sets and Systems,* vol. 347, pp. 1-28, Sep 15 2018.

[43] P. Grzegorzewski and E. Mrowka, "Trapezoidal approximations of fuzzy numbers," *Fuzzy Sets and Systems,* vol. 153, no. 1, pp. 115-135, Jul 1 2005.

[44] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, pp. 87–96, 1986.

[45] G. Li, G. Kou, and Y. Peng, "A Group Decision Making Model for Integrating Heterogeneous Information," *Ieee Transactions on Systems Man Cybernetics-Systems,* vol. 48, no. 6, pp. 982-992, Jun 2018.

[46] Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems,* vol. 35, no. 4, pp. 417-433, Aug 2006.

[47] Z. Xu, "Intuitionistic fuzzy aggregation operations," *IEEE Transactions on Fuzzy Systems* 15 (2007a) 1179–1187.

[48] Z. Xu, "Multi-person multi-attribute decision making models under intuitionistic fuzzy environment," *Fuzzy Optimization and Decision Making,* vol. 6, no. 3, pp. 221-236, Sep 2007.

[49] G. Wei, "Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making," *Applied Soft Computing,* vol. 10, no. 2, pp. 423-431, Mar 2010.

[50] C. Tan and X. Chen, "Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making," *Expert Systems with Applications,* vol. 37, no. 1, pp. 149-157, Jan 2010.

[51] D. Yu, Y. Wu, and T. Lu, "Interval-valued intuitionistic fuzzy prioritized operators and their application in group decision making," *Knowledge-Based Systems,* vol. 30, pp. 57-66, Jun 2012.

[52] X. Zhao and G. Wei, "Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making," *Knowledge-Based Systems,* vol. 37, pp. 472-479, Jan 2013.

[53] P. Liu, "Some Hamacher Aggregation Operators Based on the Interval-Valued Intuitionistic Fuzzy Numbers and Their Application to Group Decision Making," *Ieee Transactions on Fuzzy Systems,* vol. 22, no. 1, pp. 83-97, Feb 2014.

[54] D. Joshi and S. Kumar, "Interval-valued intuitionistic hesitant fuzzy Choquet integral based TOPSIS method for multi-criteria group decision making," *European Journal of Operational Research*, vol. 248, no. 1, pp. 183-191, Jan 1 2016.

[55] P. Liu, "Multiple attribute group decision making method based on

interval-valued intuitionistic fuzzy power Heronian aggregation operators," *Computers & Industrial Engineering*, vol. 108, pp. 199-212, Jun 2017.

[56] P. Liu and H. Li, "Interval-Valued Intuitionistic Fuzzy Power Bonferroni Aggregation Operators and Their Application to Group Decision Making," *Cognitive Computation*, vol. 9, no. 4, pp. 494-512, Aug 2017.

[57] G. Kaur and H. Garg, "Generalized Cubic Intuitionistic Fuzzy Aggregation Operators Using t-Norm Operations and Their Applications to Group Decision-Making Process," *Arabian Journal for Science and Engineering*, vol. 44, no. 3, pp. 2775-2794, Mar 2019.

[58] F. Teng and P. Liu, "Multiple-Attribute Group Decision-Making Method Based on the Linguistic Intuitionistic Fuzzy Density Hybrid Weighted Averaging Operator," *International Journal of Fuzzy Systems*, vol. 21, no. 1, pp. 213-231, Feb 2019.

[59] Z. Xu, "Dynamic intuitionistic fuzzy multi-attribute decision making (vol 48, pg 246, 2008)," *International Journal of Approximate Reasoning*, vol. 51, no. 1, pp. 162-164, Dec 2009.

[60] G. W. Wei, "Some geometric aggregation functions and their application to dynamic multiple attribute decision making in the intuitionisitc fuzzy setting," *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, vol. 17, no. 2, pp. 179-196, 2009.

[61] J. H. Park, H. J. Cho, and Y. C. Kwun, "Extension of the VIKOR method to dynamic intuitionistic fuzzy multiple attribute decision making," *Computers & Mathematics with Applications*, vol. 65, no. 4, pp. 731-744, Feb 2013.

[62] J. Li, W. Chen, Z. Yang, C. Li, and J. S. Sellers, "Dynamic interval-valued intuitionistic normal fuzzy aggregation operators and their applications to multi-attribute decision-making," *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 4, pp. 3937-3954, 2018.

[63] Z. Hao, Z. Xu, H. Zhao, and H. Fujita, "A Dynamic Weight Determination Approach Based on the Intuitionistic Fuzzy Bayesian Network and Its Application to Emergency Decision Making," *Ieee Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 1893-1907, Aug 2018.

[64] R. Lourenzutti, R. A. Krohling, and M. Z. Reformat, "Choquet based TOPSIS and TODIM for dynamic and heterogeneous decision making with criteria interaction," *Information Sciences*, vol. 408, pp. 41-69, Oct 2017.

[65] B. Chen, Y. Guo, X. Gao, and Y. Wang, "A Novel Multi-Attribute Decision Making Approach: Addressing the Complexity of Time Dependent and Interdependent Data," *Ieee Access*, vol. 6, pp. 55838-55849, 2018.

[66] K. Yin, P. Wang, and X. Jin, "Dynamic Intuitionistic Fuzzy Multi-Attribute Group Decision-Making Based on Power Geometric Weighted Average Operator and Prediction Model," *Symmetry-Basel*, vol. 10, no. 11, Nov 2018.

[67] Fxsjy, "Jieba," 2017, https://github.com/fxsjy/jieba.

[68] P. D. Turney, "Mining the web for synonyms: PMI-IR versus LSA on TOEFL," in: Proceeding of the 12th European Conference on Machine Learning (ECML-2001), Freiburg, Germany, 2001, pp. 491–502.

[69] L. Gao, X. Dai, S. Huang, and J. Chen, "Product Attribute Extraction Based on Feature Selection and Pointwise Mutual Information Pruning," *Pattern Recognition and Artificial Intelligence*, vol. 28, no. 2, pp. 187-192, 2015.

[70] D. Lou and T. Yao, "Semantic polarity analysis and opinion mining on Chinese review sentences," *Computer Applications*, vol. 26, no. 11, pp. 2622-2625, 2006.

[71] E. Szmidt and J. Kacprzyk, "Entropy for intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 118, no. 3, pp. 467-477, Mar 16 2001.

[72] J. F. Bard, " Managerial decisions under uncertainty: an introduction to the analysis of decision making-BAIRD,BF," *Ieee Transactions on Engineering Management*, vol. 38, no. 1, pp. 86-87, Feb 1991.